SANTIAGO ÁLVAREZ ARECES

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Matemáticas

Problemas propuestos y resueltos para: F.P. B.U.P. y C.O.U.

11

INTEGRALES INDEFINIDAS

EDITORIAL EVEREST, S. A.

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INTEGRALES INDEFINIDAS

Función primitiva e integral indefinida

Dada la función f(x), *si existe una función* F(x) *tal que en un cierto intervalo* [a, b] *sea:*

$$F'(x) = f(x)$$

se dice que F(x) *es una función primitiva de* f(x) *en ese intervalo.*

Si F(x) *es una función primitiva de* f(x), *también lo será:*

$$F(x) + C$$

Al conjunto F(x) + C *de todas las primitivas de* f(x), *se le llama integral indefinida o simplemente integral, y se representa por:*

$$\int f(x)\,dx = F(x) + C$$

Signo de integración: $\int$

Integrando: f(x)

Elemento de integración: f(x) dx

Constante de integración: C

Propiedades de la integral indefinida

1.ª $\int k\,f(x)\,dx = k\int f(x)\,dx$

2.ª $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Tabla de integrales

$$\int u^m \cdot u'\,dx = \frac{u^{m+1}}{m+1} + C \quad (m \neq -1)$$

$$\int \frac{u'\,dx}{\sqrt{u}} = 2\sqrt{u} + C$$

$$\int \frac{u'\,dx}{u} = L\,|u| + C$$

$$\int e^u \cdot u'\,dx = e^u + C$$

$$\int a^u \cdot u'\,dx = \frac{a^u}{L\,a} + C$$

$$\int \operatorname{sen} u \cdot u'\,dx = -\cos u + C$$

$$\int \cos u \cdot u' \, dx = \operatorname{sen} u + C$$

$$\int \operatorname{tg} u \cdot u' \, dx = -L\,|\cos u| + C$$

$$\int \operatorname{ctg} u \cdot u' \, dx = L\,|\operatorname{sen} u| + C$$

$$\int \sec^2 u \cdot u' \, dx = \operatorname{tg} u + C$$

$$\int \operatorname{cosec}^2 u \cdot u' \, dx = -\operatorname{ctg} u + C$$

$$\int \frac{u' \, dx}{\sqrt{1 - u^2}} = \operatorname{arc\,sen} u + C$$

$$\int \frac{u' \, dx}{1 + u^2} = \operatorname{arc\,tg} u + C$$

$$\int \frac{u' \, dx}{u\sqrt{u^2 - 1}} = \operatorname{arc\,sec} u + C$$

Integración por descomposición

La segunda propiedad de las integrales indefinidas permite descomponer una integral en suma o diferencia de varias, si el integrando se expresa como suma o diferencia de varias funciones.

Integración por sustitución o cambio de variable

A veces, para calcular:

$$\int f(x)\,dx \quad (1)$$

se efectúa un cambio de variable:

$$x = g(t) \quad \textit{siendo} \quad dx = g'(t)\,dt$$

valores que sustituidos en (1), resulta:

$$\int f(x)\,dx = \int f[g(t)]\,g'(t)\,dt = F(t) + C = F[h(x)] + C$$

siendo $t = h(x)$ *la función inversa de* $x = g(t)$

Integración por partes

Si u *y* v *son funciones continuas de* x, *se verifica:*

$$\int u\,dv = uv - \int v\,du$$

Integración de funciones racionales

Sea la integral:

$$\int \frac{P(x)}{Q(x)}\,dx$$

donde $P(x)$ *y* $Q(x)$ *son dos polinomios.*

Si el grado de $P(x) \geqslant$ *grado de* $Q(x)$, *se realiza la división de* $P(x)$ *por* $Q(x)$:

$$\begin{array}{l|l} P(x) & Q(x) \\ R(x) & C(x) \end{array} \quad \textit{de donde:}\ P(x) = Q(x) \cdot C(x) + R(x);$$

dividiendo ambos miembros por $Q(x)$:

$$\frac{P(x)}{Q(x)} = C(x) + \frac{R(x)}{Q(x)}$$

y la integral quedará:

$$\int \frac{P(x)}{Q(x)}\,dx = \int C(x)\,dx + \int \frac{R(x)}{Q(x)}\,dx$$

La primera integral del segundo miembro es inmediata, queda ahora el problema de calcular la integral:

$$\int \frac{R(x)}{Q(x)}\,dx \quad \textit{donde grado de}\ R(x) < \textit{grado de}\ Q(x)$$

Para calcular esta integral se procede de la siguiente manera:

Se iguala $Q(x) = 0$ *y se obtienen sus raíces:*

I. $Q(x) = 0$. *Sólo tiene raíces reales simples:*

$$Q(x) = (x - x_1)(x - x_2) \cdots (x - x_n)$$

Se pone:

$$\frac{R(x)}{Q(x)} = \frac{A_1}{x - x_1} + \frac{A_2}{x - x_2} + \cdots + \frac{A_n}{x - x_n} \quad (1)$$

luego: $R(x) = A_1 (x - x_2) \cdots (x - x_n) + \cdots + A_n (x - x_1) \cdots \cdots (x - x_{n-1})$

Se aplica el método de los coeficientes indeterminados para calcular $A_1, A_2, \cdots, A_n$; *se sustituyen sus valores en (1) y luego se integra:*

$$\int \frac{R(x)}{Q(x)}\,dx = A_1 \int \frac{dx}{x - x_1} + A_2 \int \frac{dx}{x - x_2} + \cdots$$

$$\cdots + A_n \int \frac{dx}{x - x_n} = A_1 \, L\,|x - x_1| + \cdots + A_n \, L|x - x_n| + C$$

II. *Q(x) = 0. Tiene raíces múltiples:*

$$Q(x) = (x - x_1)\,(x - x_1) \overset{n}{\cdots} (x - x_1) = (x - x_1)^n$$

Se descompone:

$$\frac{R(x)}{Q(x)} = \frac{A_1}{x - x_1} + \frac{A_2}{(x - x_1)^2} + \cdots + \frac{A_n}{(x - x_1)^n}$$

y luego se procede como en el caso anterior.

III. $Q(x) = 0$. *Tiene raíces imaginarias.*

Si la ecuación $Q(x) = 0$ *admite la raíz* $x_1 = \alpha + \beta i$, *admite también la conjugada* $x_2 = \alpha - \beta i$

$$Q(x) = (x - \alpha - \beta i)\,(x - \alpha + \beta i) = (x - \alpha)^2 + \beta^2$$

Se descompone:

$$\frac{R(x)}{Q(x)} = \frac{Ax + B}{(x - \alpha)^2 + \beta^2}$$

y se procede como en los casos anteriores.

Integración de funciones trigonométricas

I. *Integrales del tipo* $\int R(\operatorname{sen} x, \cos x)\,dx$; *se racionalizan mediante el cambio:*

$$\operatorname{tg}\frac{x}{2} = t \;;\; \frac{x}{2} = \operatorname{arc\,tg}\, t \;;\; x = 2 \operatorname{arc\,tg}\, t \;;\; dx = \frac{2\,dt}{1 + t^2}$$

siendo:

$$\operatorname{sen} x = \frac{2 \operatorname{tg}\frac{x}{2}}{1 + \operatorname{tg}^2\frac{x}{2}} = \frac{2t}{1 + t^2}$$

$$\cos x = \frac{1 - \operatorname{tg}^2\frac{x}{2}}{1 + \operatorname{tg}^2\frac{x}{2}} = \frac{1 - t^2}{1 + t^2}$$

II. *Integrales del tipo:* $\int \operatorname{sen}^m x \cdot \cos^n x\,dx$

Si m = impar, *se hace* $\cos x = t$

Si n = impar, *se hace* $\operatorname{sen} x = t$

Si m *y* n *tienen la misma paridad, se hace:*

$$\operatorname{tg} x = t \;;\; x = \operatorname{arc\,tg}\, t \;;\; dx = \frac{dt}{1 + t^2}, \text{ siendo:}$$

$$\operatorname{sen} x = \frac{t}{\sqrt{1 + t^2}} \;;\; \cos x = \frac{1}{\sqrt{1 + t^2}}$$

III. *Integrales del tipo:* $\int \operatorname{sen} a \cdot \cos b\,dx$; $\int \cos a \cdot \cos b\,dx$; $\int \operatorname{sen} a \cdot \operatorname{sen} b\,dx$

Teniendo en cuenta:

$$\operatorname{sen} a \cdot \cos b = \frac{1}{2}[\operatorname{sen}(a + b) + \operatorname{sen}(a - b)]$$

$$\cos a \cdot \cos b = \frac{1}{2}[\cos(a + b) + \cos(a - b)]$$

$$\operatorname{sen} a \cdot \operatorname{sen} b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$$

resulta:

$$\int \operatorname{sen} a \cdot \cos b\,dx = \frac{1}{2}\int [\operatorname{sen}(a + b) + \operatorname{sen}(a - b)]\,dx$$

$$\int \cos a \cdot \cos b\,dx = \frac{1}{2}\int [\cos(a + b) + \cos(a - b)]\,dx$$

$$\int \operatorname{sen} a \cdot \operatorname{sen} b\,dx = \frac{1}{2}\int [\cos(a - b) - \cos(a + b)]\,dx$$

IV. *Integrales del tipo:* $\int \operatorname{sen}^m x\,dx$; $\int \cos^m x\,dx$

Teniendo en cuenta:

$$\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2} \;;\; \cos^2 x = \frac{1 + \cos 2x}{2}$$

resulta:

$$\text{Si } m = \text{par} \begin{cases} \displaystyle\int \operatorname{sen}^m x\,dx = \int \left(\frac{1 - \cos 2x}{2}\right)^{m/2} dx \\ \displaystyle\int \cos^m x\,dx = \int \left(\frac{1 + \cos 2x}{2}\right)^{m/2} dx \end{cases}$$

$$\text{Si } m = \text{impar} \begin{cases} \int \text{sen}^m x\,dx = \int \text{sen}^{m-1} x \cdot \text{sen}\, x\,dx = \\ = \int (1 - \cos^2 x)^{(m-1)/2} \text{sen}\, x\,dx \\ \int \cos^m x\,dx = \int \cos^{m-1} x \cdot \cos x\,dx = \\ = \int (1 - \text{sen}^2 x)^{(m-1)/2} \cos x\,dx \end{cases}$$

Integración de funciones irracionales

I. *Integrales del tipo:* $\int R(x^{m/n}, x^{p/q}, \ldots, x^{r/s})\,dx$

Se hace: $x = t^M$ *siendo* $M = \text{m.c.m.}\,(n, q, \ldots, s)$

II. *Integrales del tipo:* $\int R\,(x, \sqrt[n]{ax + b})\,dx$

Se transforma en racional haciendo el cambio de variable:

$$ax + b = t^n$$

III. *Integrales del tipo:* $\int R\left(x, \sqrt[n]{\frac{ax + b}{cx + d}}\right) dx$

Se hace el cambio de variable: $\sqrt[n]{\frac{ax + b}{cx + d}} = t$

IV. *Integrales del tipo:*

$\int R(x, \sqrt{a^2 - x^2})\,dx$; *se hace:* $x = a \,\text{sen}\, t$ *o* $x = a \cos t$

$\int R(x, \sqrt{a^2 + x^2})\,dx$; *se hace:* $x = a \,\text{tg}\, t$

$\int R(x, \sqrt{x^2 - a^2})\,dx$; *se hace:* $x = a \sec t$

EJERCICIOS

1. Calcular: $I = \int x^3\,dx$

SOLUCIÓN: $\boxed{I = \frac{x^4}{4} + C}$

2. Calcular: $I = \int \frac{dx}{x^3}$

SOLUCIÓN: $\boxed{I = -\frac{1}{2x^2} + C}$

3. Calcular: $I = \int x \cdot \sqrt[3]{x}\,dx$

SOLUCIÓN: $\boxed{I = \frac{3}{7} x^2 \cdot \sqrt[3]{x} + C}$

4. Calcular: $I = \int \frac{dx}{\sqrt[3]{x}}$

SOLUCIÓN: $\boxed{I = \frac{3}{2} \cdot \sqrt[3]{x^2} + C}$

5. Calcular: $I = \int \left(\frac{2}{3} x^4 - \frac{2}{4} x^2 + 1\right) dx$

SOLUCIÓN: $\boxed{I = \frac{2x^5}{15} - \frac{x^3}{6} + x + C}$

6. Calcular: $I = \int \left(\frac{2}{3} x^{3/2} - \frac{3}{4} x^{1/2} - \frac{2}{3}\right) dx$

SOLUCIÓN: $\boxed{I = \frac{2x^{5/3}}{5} - \frac{x^{3/2}}{2} - \frac{2x}{3} + C}$

7. Calcular: $I = \int \left(-\frac{3}{x^4} - \frac{1}{x^3} + \frac{2}{x^5} \right) dx$

SOLUCIÓN: $\boxed{I = \frac{1}{x^3} + \frac{1}{2x^2} - \frac{1}{2x^4} + C}$

8. Calcular: $I = \int (x^2 - 3x + 4)\, dx$

SOLUCIÓN: $\boxed{I = \frac{x^3}{3} - \frac{3x^2}{2} + 4x + C}$

9. Calcular: $I = \int \frac{2x^3 + 2x + 1}{1 + x^2} dx$

SOLUCIÓN: $\boxed{I = x^2 + \text{arc tg } x + C}$

10. Calcular: $I = \int \frac{x - 1}{x + 1} dx$

SOLUCIÓN: $\boxed{I = x - 2\,L\,|x + 1| + C}$

11. Calcular: $I = \int \frac{x^4 - 5x^2 + 10}{x^2} dx$

SOLUCIÓN: $\boxed{I = \frac{x^3}{3} - 5x - \frac{10}{x} + C}$

12. Calcular: $I = \int (\sqrt{a} - \sqrt{x})^2\, dx$

SOLUCIÓN: $\boxed{I = ax - \frac{4x\sqrt{ax}}{3} + \frac{x^2}{2} + C}$

13. Calcular: $I = \int \sqrt{x}\,(\sqrt{a} - \sqrt{x})^2\, dx$

SOLUCIÓN: $\boxed{I = \frac{2ax^{3/2}}{3} - x^2\sqrt{a} + \frac{2x^{5/2}}{5} + C}$

14. Calcular: $I = \int 10^x\, dx$

SOLUCIÓN: $\boxed{I = \frac{10^x}{L\,10} + C}$

15. Calcular: $I = \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$

SOLUCIÓN: $\boxed{I = \frac{x^2}{2} + 2x + L\,|x| + C}$

16. Calcular: $I = \int \frac{dx}{\text{sen}^2 x \cdot \cos^2 x}$

SOLUCIÓN: $\boxed{I = \text{tg } x - \text{ctg } x + C}$

17. Calcular: $I = \int \text{tg}^2 x\, dx$

SOLUCIÓN: $\boxed{I = \text{tg } x - x + C}$

18. Calcular: $I = \int \frac{3\cos x + 2 - 2\,\text{sen}^2 x}{\cos x} dx$

SOLUCIÓN: $\boxed{I = 3x + 2\,\text{sen } x + C}$

19. Calcular: $I = \int \frac{(\sqrt{a} + \sqrt{x})^2}{\sqrt{x}} dx$

SOLUCIÓN: $\boxed{I = 2a\sqrt{x} + 2x\sqrt{a} + \frac{2}{3} x\sqrt{x} + C}$

20. Calcular: $I = \int \frac{x^3 - 6x + 5}{x - 2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{x^3}{3} + x^2 - 2x + L\,|x - 2| + C}$

21. Calcular: $I = \int \left(\frac{\text{sen } 2x}{\text{sen } x \cos x} + \cos x \right) dx$

SOLUCIÓN: $\boxed{I = 2x + \text{sen } x + C}$

22. Calcular: $I = \int (4x - 2)^5\, dx$

SOLUCIÓN: $\boxed{I = \frac{(4x - 2)^6}{24} + C}$

23. Calcular: $I = \int x\,(3x^2 + 1)\, dx$

SOLUCIÓN: $\boxed{I = \frac{(3x^2 + 1)^2}{12} + C}$

24. Calcular: $I = \int \frac{2x + 1}{x^2 + x - 3}\, dx$

SOLUCIÓN: $\boxed{I = L\,|x^2 + x - 3| + C}$

25. Calcular: $I = \int (x^3 - 5x^2 + 4x)\,(3x^2 - 10x + 4)\, dx$

SOLUCIÓN: $\boxed{I = \frac{(x^3 - 5x^2 + 4x)^2}{2} + C}$

26. Calcular: $I = \int 2x\sqrt{1 + 3x^2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{2}{9}(1 + 3x^2)\sqrt{(1 + 3x^2)} + C}$

27. Calcular: $I = \int \frac{2x}{\sqrt{8 + x^2}}\, dx$

SOLUCIÓN: $\boxed{I = 2\sqrt{8 + x^2} + C}$

28. Calcular: $I = \int (x + 3)\,(x^2 + 6x - 4)\, dx$

SOLUCIÓN: $\boxed{I = \frac{(x^2 + 6x - 4)^2}{4} + C}$

29. Calcular: $I = \int (x + 3)\,\text{sen}\,(x^2 + 6x - 4)\, dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{2}\cos\,(x^2 + 6x - 4) + C}$

30. Calcular: $I = \int x\sqrt{x - 1}\, dx$

SOLUCIÓN: $\boxed{I = \frac{2(x - 1)^{5/2}}{5} + \frac{2(x - 1)^{3/2}}{3} + C}$

31. Calcular: $I = \int x \cdot \text{sen } x^2\, dx$

SOLUCIÓN: $\boxed{I = -\frac{\cos x^2}{2} + C}$

32. Calcular: $I = \int \frac{dx}{\sqrt{x}\cos^2\sqrt{x}}$

SOLUCIÓN: $\boxed{I = 2\,tg\sqrt{x} + C}$

33. Calcular: $I = \int \frac{x}{(x+1)(x-1)}\,dx$

SOLUCIÓN: $\boxed{I = \frac{L\,|x^2-1|}{2} + C}$

34. Calcular: $I = \int \frac{dx}{\sqrt{7x-2}}$

SOLUCIÓN: $\boxed{I = \frac{2\sqrt{7x-2}}{7} + C}$

35. Calcular: $I = \int \frac{arc\ sen\ x}{\sqrt{1-x^2}}\,dx$

SOLUCIÓN: $\boxed{I = \frac{(arc\ sen\ x)^2}{2} + C}$

36. Calcular: $I = \int x\sqrt{5x^2+1}\,dx$

SOLUCIÓN: $\boxed{I = \frac{(5x^2+1)^{3/2}}{15} + C}$

37. Calcular: $I = \int \frac{L\,x}{x}\,dx$

SOLUCIÓN: $\boxed{I = \frac{(L\,x)^2}{2} + C}$

38. Calcular: $I = \int \frac{dx}{(x-1)^2}$

SOLUCIÓN: $\boxed{I = -\frac{1}{x-1} + C}$

39. Calcular: $I = \int \frac{(L\,x)^3}{x}\,dx$

SOLUCIÓN: $\boxed{I = \frac{(L\,x)^4}{4} + C}$

40. Calcular: $I = \int \frac{dx}{(1+x)\sqrt{x}}$

SOLUCIÓN: $\boxed{I = 2\,arc\,tg\sqrt{x} + C}$

41. Calcular: $I = \int \frac{x^2}{\sqrt{5x^3+7}}\,dx$

SOLUCIÓN: $\boxed{I = \frac{2}{15}\sqrt{5x^3+7} + C}$

42. Calcular: $I = \int \frac{6x^3-11x^2-19x-7}{3x+2}\,dx$

SOLUCIÓN: $\boxed{I = \frac{2x^3}{3} - \frac{5x^2}{2} - 3x - \frac{1}{3}L\,|3x+2| + C}$

43. Calcular: $I = \int \frac{\sqrt{x}}{\sqrt{x}-2}\,dx$

SOLUCIÓN: $\boxed{I = x + 4\sqrt{x} + 8\,L\,|\sqrt{x}-2| + C}$

44. Calcular: $I = \int \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}\, dx$

SOLUCIÓN: $\boxed{I = x - \frac{3}{2}\sqrt[3]{x^2} + 3\sqrt[3]{x} - 3L\left|\sqrt[3]{x} + 1\right| + C}$

45. Calcular: $I = \int (e^x - 3e^{2x} + 4e^{3x})\, dx$

SOLUCIÓN: $\boxed{I = e^x - \frac{3}{2}e^{2x} + \frac{4}{3}e^{3x} + C}$

46. Calcular: $I = \int \operatorname{sen}^3 3x \cdot \cos 3x\, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{sen}^4 3x}{12} + C}$

47. Calcular: $I = \int \frac{e^x}{e^x + 2}\, dx$

SOLUCIÓN: $\boxed{I = L\left|e^x + 2\right| + C}$

48. Calcular: $I = \int \frac{2x}{1 + \sqrt{x}}\, dx$

SOLUCIÓN: $\boxed{I = \frac{4}{3}x\sqrt{x} - 2x + 4\sqrt{x} - 4L\left|\sqrt{x} + 1\right| + C}$

49. Calcular: $I = \int \frac{\operatorname{ctg} x}{\sqrt{\operatorname{sen} x}}\, dx$

SOLUCIÓN: $\boxed{I = -\frac{2}{\sqrt{\operatorname{sen} x}} + C}$

50. Calcular: $I = \int \frac{dx}{\sqrt{x}}$

SOLUCIÓN: $\boxed{I = 2\sqrt{x} + C}$

51. Calcular: $I = \int \frac{dx}{\sqrt[3]{x}}$

SOLUCIÓN: $\boxed{I = \frac{3\sqrt[3]{x^2}}{2} + C}$

52. Calcular: $I = \int \sqrt{ax}\, dx$

SOLUCIÓN: $\boxed{I = \frac{2x\sqrt{ax}}{3} + C}$

53. Calcular: $I = \int \frac{dx}{\sqrt{a - bx}}$

SOLUCIÓN: $\boxed{I = -\frac{2\sqrt{a - bx}}{b} + C}$

54. Calcular: $I = \int \frac{dx}{\sqrt{9 - 16x^2}}$

SOLUCIÓN: $\boxed{I = \frac{1}{4}\operatorname{arc\,sen}\frac{4x}{3} + C}$

55. Calcular: $I = \int \sqrt{a + bx}\, dx$

SOLUCIÓN: $\boxed{I = \frac{2(a + bx)^{3/2}}{3b} + C}$

56. Calcular: $I = \int x\,(2 + x^2)^2\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{(2 + x^2)^3}{6} + C}$

57. Calcular: $I = \int x\sqrt{2x^2 + 3}\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{(2x^2 + 3)^{3/2}}{6} + C}$

58. Calcular: $I = \int \dfrac{4x^2}{\sqrt{x^3 + 8}}\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{8\sqrt{x^3 + 8}}{3} + C}$

59. Calcular: $I = \int \dfrac{(\sqrt{a} + \sqrt{x})^2}{\sqrt{x}}\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{2\,(\sqrt{a} + \sqrt{x})^3}{3} + C}$

60. Calcular: $I = \int \dfrac{x^3}{\sqrt{a^4 - x^4}}\,dx$

SOLUCIÓN: $\boxed{I = -\dfrac{\sqrt{a^4 - x^4}}{2} + C}$

61. Calcular: $I = \int \dfrac{x}{(a + bx^2)^3}\,dx$

SOLUCIÓN: $\boxed{I = -\dfrac{1}{4b\,(a + bx^2)^2} + C}$

62. Calcular: $I = \int x^{n-1} \cdot \sqrt{a + bx^n}\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{2\,(a + bx^n)^{3/2}}{3bn} + C}$

63. Calcular: $I = \int \dfrac{2x + 3}{\sqrt{x^2 + 3x - 1}}\,dx$

SOLUCIÓN: $\boxed{I = 2\sqrt{x^2 + 3x - 1} + C}$

64. Calcular: $I = \int \dfrac{x^2 + 1}{\sqrt{x^3 + 3x}}\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{2\sqrt{x^3 + 3x}}{3} + C}$

65. Calcular: $I = \int \dfrac{x + 5}{\sqrt{x^2 + 10x - 4}}\,dx$

SOLUCIÓN: $\boxed{I = \sqrt{x^2 + 10 - 4} + C}$

66. Calcular: $I = \int \dfrac{2 + Lx}{x}\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{(2 + Lx)^2}{2} + C}$

67. Calcular: $I = \int \operatorname{sen} 3x \cdot \cos 3x\,dx$

SOLUCIÓN: $\boxed{I = \dfrac{\operatorname{sen}^2 3x}{6} + C}$

68. Calcular: $I = \int (\operatorname{sen} 2x \cdot \cos^2 2x)\, dx$

SOLUCIÓN: $\boxed{I = -\frac{\cos^3 2x}{6} + C}$

69. Calcular: $I = \int \operatorname{tg} \frac{x}{2} \cdot \sec^2 \frac{x}{2}\, dx$

SOLUCIÓN: $\boxed{I = \operatorname{tg}^2 \frac{x}{2} + C}$

70. Calcular: $I = \int \frac{\cos 4x}{\sqrt{3 + \operatorname{sen} 4x}}\, dx$

SOLUCIÓN: $\boxed{I = \frac{\sqrt{3 + \operatorname{sen} 4x}}{2} + C}$

71. Calcular: $I = \int \left(\frac{\sec 2x}{1 + \operatorname{tg} 2x}\right)^2 dx$

SOLUCIÓN: $\boxed{I = \frac{-1}{2(1 + \operatorname{tg} 2x)} + C}$

72. Calcular: $I = \int \frac{e^{2x}}{3 + 5e^{2x}}\, dx$

SOLUCIÓN: $\boxed{I = \frac{L\,|3 + 5e^{2x}|}{10} + C}$

73. Calcular: $I = \int \frac{x}{\sqrt{1 - 2x^2}}\, dx$

SOLUCIÓN: $\boxed{I = -\frac{\sqrt{1 - 2x^2}}{2} + C}$

74. Calcular: $I = \int \frac{\operatorname{sen} ax}{\cos ax + b}\, dx$

SOLUCIÓN: $\boxed{I = -\frac{L\,|\cos ax + b|}{a} + C}$

75. Calcular: $I = \int \frac{\operatorname{cosec}^2 x}{\sqrt{2 \operatorname{ctg} x + 3}}\, dx$

SOLUCIÓN: $\boxed{I = -\sqrt{2 \operatorname{ctg} x + 3} + C}$

76. Calcular: $I = \int \frac{e^x + 2}{\sqrt{e^x + 2x}}\, dx$

SOLUCIÓN: $\boxed{I = 2\sqrt{e^x + 2x} + C}$

77. Calcular: $I = \int \frac{e^x + \operatorname{sen} x}{\sqrt{e^x - \cos x}}\, dx$

SOLUCIÓN: $\boxed{I = 2\sqrt{e^x - \cos x} + C}$

78. Calcular: $I = \int \frac{\sec 2x \cdot \operatorname{tg} 2x}{\sec 2x - 3}\, dx$

SOLUCIÓN: $\boxed{I = \frac{L\,|\sec 2x - 3|}{2} + C}$

79. Calcular: $I = \int (e^{x/a} - e^{-x/a})\, dx$

SOLUCIÓN: $\boxed{I = a\,(e^{x/a} + e^{-x/a}) + C}$

80. Calcular: $I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

SOLUCIÓN: $\boxed{I = 2e^{\sqrt{x}} + C}$

81. Calcular: $I = \int e^{tg\,x} \cdot \sec^2 x\,dx$

SOLUCIÓN: $\boxed{I = e^{tg\,x} + C}$

82. Calcular: $I = \int a^{2x}\,dx$

SOLUCIÓN: $\boxed{I = \frac{a^{2x}}{2\,La} + C}$

83. Calcular: $I = \int (e^{5x} + a^{5x})\,dx$

SOLUCIÓN: $\boxed{I = \frac{e^{5x}}{5} + \frac{a^{5x}}{5\,La} + C}$

84. Calcular: $I = \int \frac{3\,dx}{e^{3x}}$

SOLUCIÓN: $\boxed{I = -e^{-3x} + C}$

85. Calcular: $I = \int 6x\,e^{-x^2}\,dx$

SOLUCIÓN: $\boxed{I = -3e^{-x^2} + C}$

86. Calcular: $I = \int \frac{e^{\sqrt{x}} - 3}{\sqrt{x}}\,dx$

SOLUCIÓN: $\boxed{I = 2\,e^{\sqrt{x}} - 6\sqrt{x} + C}$

87. Calcular: $I = \int \text{sen}\,\frac{2x}{3}\,dx$

SOLUCIÓN: $\boxed{I = -\frac{3}{2}\cos\frac{2x}{3} + C}$

88. Calcular: $I = \int \cos 5x\,dx$

SOLUCIÓN: $\boxed{I = \frac{\text{sen } 5x}{5} + C}$

89. Calcular: $I = \int \text{tg}\,\frac{x}{5}\,dx$

SOLUCIÓN: $\boxed{I = -5\,L\left|\cos\frac{x}{5}\right| + C}$

90. Calcular: $I = \int \text{ctg } 10x\,dx$

SOLUCIÓN: $\boxed{I = \frac{1}{10}\,L\,|\text{sen } 10x| + C}$

91. Calcular: $I = \int \text{cosec } x\,dx$

SOLUCIÓN: $\boxed{I = -L\,|\text{cosec } x + \text{ctg } x| + C}$

92. Calcular: $I = \int \frac{dx}{(1 + x^2)\,\text{arc tg } x}$

SOLUCIÓN: $\boxed{I = L\,|\text{arc tg } x| + C}$

93. Calcular: $I = \int \sqrt{\frac{1 + x}{1 - x}}\,dx$

SOLUCIÓN: $\boxed{I = \text{arc sen } x - \sqrt{1 - x^2} + C}$

94. Calcular: $I = \int \frac{dx}{x^2 + 9}$

SOLUCIÓN: $\boxed{I = \frac{1}{3} \operatorname{arc\,tg} \frac{x}{3} + C}$

95. Calcular: $I = \int \frac{dx}{(x - 2)^2 + 9}$

SOLUCIÓN: $\boxed{I = \frac{1}{3} \operatorname{arc\,tg} \frac{x - 2}{3} + C}$

96. Calcular: $I = \int \frac{ax}{x^4 + b^4} dx$

SOLUCIÓN: $\boxed{I = \frac{a}{2b^2} \operatorname{arc\,tg} \frac{x^2}{b^2} + C}$

97. Calcular: $I = \int \frac{dx}{x\,Lx}$

SOLUCIÓN: $\boxed{I = L\,(Lx) + C}$

98. Calcular: $I = \int \cos^2 5x\, dx$

SOLUCIÓN: $\boxed{I = \frac{x}{2} + \frac{\operatorname{sen} 10x}{20} + C}$

99. Calcular: $I = \int \cos x \cdot \operatorname{sen} 2x\, dx$

SOLUCIÓN: $\boxed{I = -\frac{2\cos^3 x}{3} + C}$

100. Calcular: $I = \int \frac{dx}{1 + \cos x}$

SOLUCIÓN: $\boxed{I = -\operatorname{ctg} x + \frac{1}{\operatorname{sen} x} + C}$

101. Calcular: $I = \int \operatorname{sen}^2 3x\, dx$

SOLUCIÓN: $\boxed{I = \frac{x}{2} - \frac{1}{12} \operatorname{sen} 6x + C}$

102. Calcular: $I = \int \operatorname{tg}^2 2x\, dx$

SOLUCIÓN: $\boxed{I = \frac{1}{2} \operatorname{tg} 2x - x + C}$

103. Calcular: $I = \int \sec^2 10x\, dx$

SOLUCIÓN: $\boxed{I = \frac{1}{10} \operatorname{tg} 10x + C}$

104. Calcular: $I = \int \frac{dx}{\sqrt{25 - x^2}}$

SOLUCIÓN: $\boxed{I = \operatorname{arc\,sen} \frac{x}{5} + C}$

105. Calcular: $I = \int \frac{5x}{\sqrt{1 - x^4}} dx$

SOLUCIÓN: $\boxed{I = \frac{5}{2} \operatorname{arc\ sen} x^2 + C}$

106. Calcular: $I = \int \sec^2\left(\frac{8x^2 - 2x - 15}{4x + 5}\right) dx$

SOLUCIÓN: $\boxed{I = \frac{1}{2}\,\mathbf{tg\,(2x - 3)} + C}$

107. Calcular: $I = \int \frac{1}{4}\, x\, a^{x^2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{a^{x^2}}{8\,La} + C}$

108. Calcular: $I = \int \frac{3\,dx}{5^{2x-1}}$

SOLUCIÓN: $\boxed{I = -\frac{3 \cdot 5^{1-2x}}{2\,L5} + C}$

109. Calcular: $I = \int \frac{x^6}{\cos^2(x^7 + 2)} dx$

SOLUCIÓN: $\boxed{I = \frac{1}{7}\,\mathbf{tg\,(x^7 + 2)} + C}$

110. Calcular: $I = \int \frac{\sqrt{x^2 - 1}}{x} dx$

SOLUCIÓN: $\boxed{I = \sqrt{x^2 - 1} - \mathbf{arc\,sec\,x} + C}$

111. Calcular: $I = \int \frac{x \cdot a^{\sqrt{x^2-1}}}{\sqrt{x^2 - 1}} dx$

SOLUCIÓN: $\boxed{I = \frac{a^{\sqrt{x^2-1}}}{La} + C}$

112. Calcular: $I = \int \frac{dx}{\sqrt{4 - (x + 3)^2}}$

SOLUCIÓN: $\boxed{I = \mathbf{arc\,sen}\,\frac{x + 3}{2} + C}$

113. Calcular: $I = \int \frac{dx}{x^2 + 2x + 5}$

SOLUCIÓN: $\boxed{I = \frac{1}{2}\,\mathbf{arc\,tg}\,\frac{x + 1}{2} + C}$

114. Calcular: $I = \int \frac{dx}{x^2 + 64}$

SOLUCIÓN: $\boxed{I = \frac{1}{8}\,\mathbf{arc\,tg}\,\frac{x}{8} + C}$

115. Calcular: $I = \int \frac{dx}{\sqrt{16 - 9x^2}}$

SOLUCIÓN: $\boxed{I = \frac{1}{3}\,\mathbf{arc\,sen}\,\frac{3x}{4} + C}$

116. Calcular: $I = \int \frac{e^x}{1 + e^{2x}} dx$

SOLUCIÓN: $\boxed{I = \mathbf{arc\,tg}\,e^x + C}$

117. Calcular: $I = \int \frac{dx}{\sqrt{1 - a^2 x^2}}$

SOLUCIÓN: $\boxed{I = \frac{1}{a}\,\mathbf{arc\,sen\,ax} + C}$

118. Calcular: $I = \int \frac{dx}{x\sqrt{1-(Lx)^2}}$

SOLUCIÓN: $\boxed{I = \text{arc sen } (Lx) + C}$

119. Calcular: $I = \int \frac{dx}{\sqrt{2x - x^2}}$

SOLUCIÓN: $\boxed{I = \text{arc sen } (x - 1) + C}$

120. Calcular: $I = \int \frac{dx}{\sqrt{1 + 4x - x^2}}$

SOLUCIÓN: $\boxed{I = \text{arc sen } \frac{x-2}{\sqrt{5}} + C}$

121. Calcular: $I = \int \frac{dx}{4x^2 + 25}$

SOLUCIÓN: $\boxed{I = \frac{1}{10} \text{arc tg } \frac{2x}{5} + C}$

122. Calcular: $I = \int \frac{dx}{3 + 7x^2}$

SOLUCIÓN: $\boxed{I = \frac{7\sqrt{3}}{21} \text{arc tg } \frac{\sqrt{7}}{\sqrt{3}} x + C}$

123. Calcular: $I = \int \frac{3\,dx}{x^2 - 8x + 25}$

SOLUCIÓN: $\boxed{I = \text{arc tg } \frac{x-4}{3} + C}$

124. Calcular: $I = \int \frac{2x + 5}{x^2 + 2x + 5} dx$

SOLUCIÓN: $\boxed{I = L\,|x^2 + 2x + 5| + \frac{3}{2} \text{arc tg } \frac{x+1}{2} + C}$

125. Calcular: $I = \int \frac{8x - 3}{\sqrt{12x - 4x^2 - 5}} dx$

SOLUCIÓN: $\boxed{I = -2\sqrt{12x - 4x^2 - 5} + \frac{9}{2} \text{arc sen } \frac{2x-3}{2} + C}$

126. Calcular: $I = \int \sec x\,dx$

SOLUCIÓN: $\boxed{I = L\,|\sec x + \text{tg } x| + C}$

127. Calcular: $I = \int \frac{\text{cosec } 2x \cdot \text{ctg } 2x}{5 - 4 \text{ cosec } 2x} dx$

SOLUCIÓN: $\boxed{I = \frac{1}{8} L\,|5 - 4 \text{ cosec } 2x| + C}$

128. Calcular: $I = \int \frac{\text{cosec}^2 \frac{x}{2}}{\sqrt{3 - \text{ctg } \frac{x}{2}}} dx$

SOLUCIÓN: $\boxed{I = 4\sqrt{3 - \text{ctg } \frac{x}{2}} + C}$

129. Calcular: $I = \int \frac{dx}{\text{sen } x \cos x}$

SOLUCIÓN: $\boxed{I = L\,|\text{tg } x| + C}$

130. Calcular: $I = \int x\, e^{-x}\, dx$

SOLUCIÓN: $\boxed{I = -e^{-x}(x + 1) + C}$

131. Calcular: $I = \int \frac{L\,x}{x^2}\, dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{x}(1 + L\,x) + C}$

132. Calcular: $I = \int x \cos x\, dx$

SOLUCIÓN: $\boxed{I = x \text{ sen } x + \cos x + C}$

133. Calcular: $I = \int x\, e^{ax}\, dx$

SOLUCIÓN: $\boxed{I = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right) + C}$

134. Calcular: $I = \int x^2\, e^{ax}\, dx$

SOLUCIÓN: $\boxed{I = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right) + C}$

135. Calcular: $I = \int L\,4x\, dx$

SOLUCIÓN: $\boxed{I = x\,(L\,4x - 1) + C}$

136. Calcular: $I = \int x \cos 4x\, dx$

SOLUCIÓN: $\boxed{I = \frac{x}{4} \text{ sen } 4x + \frac{1}{16} \cos 4x + C}$

137. Calcular: $I = \int x \sec^2 3x\, dx$

SOLUCIÓN: $\boxed{I = \frac{x \text{ tg } 3x}{3} + \frac{1}{9} L\,|\cos 3x| + C}$

138. Calcular: $I = \int \text{arc cos } x\, dx$

SOLUCIÓN: $\boxed{I = x \text{ arc cos } x - \sqrt{1 - x^2} + C}$

139. Calcular: $I = \int \text{arc tg } x\, dx$

SOLUCIÓN: $\boxed{I = x \text{ arc tg } x - \frac{1}{2} L\,|1 + x^2| + C}$

140. Calcular: $I = \int \text{arc ctg } 3x\, dx$

SOLUCIÓN: $\boxed{I = x \text{ arc ctg } x + \frac{1}{2} L\,|1 + (3x)^2| + C}$

141. Calcular: $I = \int x^2\, Lx\, dx$

SOLUCIÓN: $\boxed{I = \frac{x^3}{3}\left(Lx - \frac{1}{3}\right) + C}$

142. Calcular: $I = \int \text{sen}^2\, dx$

SOLUCIÓN: $\boxed{I = \frac{1}{2}(x - \text{sen } x \cos x) + C}$

143. Calcular: $I = \int x^2 \cos x \, dx$

SOLUCIÓN: $\boxed{I = x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C}$

144. Calcular: $I = \int x \operatorname{arc\,tg} x \, dx$

SOLUCIÓN: $\boxed{I = \frac{x^2 + 1}{2} \operatorname{arc\,tg} x - \frac{x}{2} + C}$

145. Calcular: $I = \int x^2 \cdot e^{-x} \, dx$

SOLUCIÓN: $\boxed{I = -e^{-x}(x^2 + 2x + 2) + C}$

146. Calcular: $I = \int (Lx)^2 \, dx$

SOLUCIÓN: $\boxed{I = x\,(Lx)^2 - 2x\,Lx + 2x + C}$

147. Calcular: $I = \int x \operatorname{sen} x \cdot \cos x \, dx$

SOLUCIÓN: $\boxed{I = -\frac{x \cos 2x}{4} + \frac{\operatorname{sen} 2x}{8} + C}$

148. Calcular: $I = \int \cos x \cdot L \operatorname{sen} x \, dx$

SOLUCIÓN: $\boxed{I = \operatorname{sen} x\,(L \operatorname{sen} x - 1) + C}$

149. Calcular: $I = \int e^x \cdot \cos x \, dx$

SOLUCIÓN: $\boxed{I = \frac{e^x}{2}(\cos x + \operatorname{sen} x) + C}$

150. Calcular: $I = \int \frac{Lx}{(x+1)^2} \, dx$

SOLUCIÓN: $\boxed{I = -\frac{Lx}{x+1} + L\frac{x}{x+1} + C}$

151. Calcular: $I = \int \frac{dx}{(x^2+1)^2}$

SOLUCIÓN: $\boxed{I = \frac{1}{2} \operatorname{arc\,tg} x + \frac{x}{2(x^2+1)} + C}$

152. Calcular: $I = \int \sec^3 x \, dx$

SOLUCIÓN: $\boxed{I = \frac{1}{2}[\sec x \cdot \operatorname{tg} x + L\,|\sec x + \operatorname{tg} x|] + C}$

153. Calcular: $I = \int (x^2 - 2x + 1)\,Lx \, dx$

SOLUCIÓN: $\boxed{I = \left(\frac{x^3}{3} - x^2 + x\right) Lx - \frac{x^3}{9} + \frac{x^2}{2} - x + C}$

154. Calcular: $I = \int (3x^2 - x + 5) \operatorname{sen} x \, dx$

SOLUCIÓN: $\boxed{I = -(3x^2 - x + 5)\cos x + (6x - 1)\operatorname{sen} x + 6\cos x + C}$

155. Calcular: $I = \int x^3 (Lx)^2 \, dx$

SOLUCIÓN: $\boxed{I = \frac{x^4}{4}(Lx)^2 - \frac{x^4}{8} Lx + \frac{x^4}{32} + C}$

156. Calcular: $I = \int L\,(x+1) \, dx$

SOLUCIÓN: $\boxed{I = x\,L\,|x+1| - x + L\,|x+1| + C}$

157. Calcular: $I = \int \frac{x\,dx}{\operatorname{sen}^2 x}$

SOLUCIÓN: $\boxed{I = -x\,\operatorname{ctg} x + L\,|\operatorname{sen} x| + C}$

158. Calcular: $I = \int \operatorname{sen} Lx\,dx$

SOLUCIÓN: $\boxed{I = \frac{x}{2}(\operatorname{sen} Lx - \cos Lx) + C}$

159. Calcular: $I = \int \frac{Lx}{\sqrt{x}}\,dx$

SOLUCIÓN: $\boxed{I = 2\sqrt{x}\,(Lx - 2) + C}$

160. Calcular: $I = \int e^{\operatorname{arc\,sen} x}\,dx$

SOLUCIÓN: $\boxed{I = \frac{e^{\operatorname{arc\,sen} x}}{2}(x + \sqrt{1 - x^2}) + C}$

161. Calcular: $I = \int x \operatorname{arc\,tg} x^2\,dx$

SOLUCIÓN: $\boxed{I = \frac{x^2}{2}\operatorname{arc\,tg} x^2 - \frac{1}{4} L\,|1 + x^4| + C}$

162. Calcular: $I = \int \frac{x \cdot \operatorname{arc\,sen} x}{\sqrt{1 - x^2}}\,dx$

SOLUCIÓN: $\boxed{I = -\sqrt{1 - x^2} \cdot \operatorname{arc\,sen} x + x + C}$

163. Calcular: $I = \int \frac{x\,e^x}{(1 + x)^2}\,dx$

SOLUCIÓN: $\boxed{I = \frac{e^x}{2}\left[L\,|1 + x| + \frac{1}{1 + x}\right] + C}$

164. Calcular: $I = \int \operatorname{arc}\cos 2x\,dx$

SOLUCIÓN: $\boxed{I = x \operatorname{arc}\cos 2x - \frac{1}{2}\sqrt{1 - 4x^2} + C}$

165. Calcular: $I = \int \operatorname{arc\,tg} \sqrt{x}\,dx$

SOLUCIÓN: $\boxed{I = (x + 1) \operatorname{arc\,tg} \sqrt{x} - \sqrt{x} + C}$

166. Calcular: $I = \int \frac{L(x + 1)}{\sqrt{x + 1}}\,dx$

SOLUCIÓN: $\boxed{I = 2\sqrt{x + 1}\,[L(x + 1) - 2] + C}$

167. Calcular: $I = \int x^2 \cdot \operatorname{arc\,sen} x\,dx$

SOLUCIÓN: $\boxed{I = \frac{x^3}{3}\operatorname{arc\,sen} x + \frac{\sqrt{1 - x^2}}{3} - \frac{(1 - x^2)\sqrt{1 - x^2}}{9} + C}$

168. Calcular: $I = \int x\,[L(1 + x^2) + e^{-x}]\,dx$

SOLUCIÓN: $\boxed{I = -\frac{x^2}{2} + \frac{x^2 + 1}{2} L(1 + x^2) - e^{-x}(x + 1) + C}$

169. Calcular: $I = \int \frac{dx}{(x - 1)(x + 1)}$

SOLUCIÓN: $\boxed{I = L\left|\frac{\sqrt{x - 1}}{\sqrt{x + 1}}\right| + C}$

170. Calcular: $I = \int \frac{x}{x^2 - x - 2}\, dx$

SOLUCIÓN: $\boxed{I = L\left|\sqrt[3]{(x-2)^2\,(x+1)}\right| + C}$

171. Calcular: $I = \int \frac{dx}{x^2 - 9}$

SOLUCIÓN: $\boxed{I = L\left|\sqrt[6]{\frac{x-3}{x+3}}\right| + C}$

172. Calcular: $I = \int \frac{dx}{x\,(x-1)\,(x-2)}$

SOLUCIÓN: $\boxed{I = L\left|\frac{\sqrt{x\,(x-2)}}{x-1}\right| + C}$

173. Calcular: $I = \int \frac{x^3}{x^2 - x - 2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{x^2}{2} + x + \frac{8}{3} L\,|x-2| + \frac{1}{3} L\,|x-1| + C}$

174. Calcular: $I = \int \frac{4x - 2}{x^3 - x^2 - 2x}\, dx$

SOLUCIÓN: $\boxed{I = L\left|\frac{x\,(x-2)}{x+1}\right| + C}$

175. Calcular: $I = \int \frac{5x^2 - 3}{x^3 - x}\, dx$

SOLUCIÓN: $\boxed{I = L\,|x^3\,(x^2-1)| + C}$

176. Calcular: $I = \int \frac{(4x^3 + 2x^2 + 1)}{4x^3 - x}\, dx$

SOLUCIÓN: $\boxed{I = x + L\left|\frac{(2x-1)\sqrt{2x+1}}{x}\right| + C}$

177. Calcular: $I = \int \frac{(3x^2 + 5x)}{(x-1)\,(x+1)^2}\, dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{x+1} + L\,|(x-1)^2\,(x+1)| + C}$

178. Calcular: $I = \int \frac{x^2}{(x+1)^3}\, dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{2\,(x+1)^2} + \frac{2}{x+1} + L\,|x+1| + C}$

179. Calcular: $I = \int \frac{dx}{(1+x)\,(1-x^2)}$

SOLUCIÓN: $\boxed{I = -\frac{1}{2\,(1+x)} + L\left|\sqrt[4]{\frac{1+x}{1-x}}\right| + C}$

180. Calcular: $I = \int \frac{2x + 3}{x^3 + x^2 - 2x}\, dx$

SOLUCIÓN: $\boxed{I = L\left|\frac{(x-1)^{5/3}}{x^{3/2}\,(x+2)^{1/6}}\right| + C}$

181. Calcular: $I = \int \frac{(x^3 + 1)}{x\,(x-1)^3}\, dx$

SOLUCIÓN: $\boxed{I = L\left|\frac{(x-1)^2}{x}\right| - \frac{x}{(x-1)^2} + C}$

182. Calcular: $I = \int \frac{x^2}{(x-1)^3}\,dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{2(x-1)^2} - \frac{2}{x-1} + L\,|x-1| + C}$

183. Calcular: $I = \int \frac{x^4 - 8}{x^3 + 2x^2}\,dx$

SOLUCIÓN: $\boxed{I = \frac{x^2}{2} - 2x + \frac{4}{x} + L\,|x^2\,(x+2)^2| + C}$

184. Calcular: $I = \int \frac{8}{x^3 - 4x}\,dx$

SOLUCIÓN: $\boxed{I = L\left|\frac{x^2 - 4}{x^2}\right| + C}$

185. Calcular: $I = \int \frac{3x^2 + 11x + 2}{(x+3)\,(x^2-1)}\,dx$

SOLUCIÓN: $\boxed{I = L\left|\frac{(x-1)^2 \cdot \sqrt{(x+1)^3}}{\sqrt{x+3}}\right| + C}$

186. Calcular: $I = \int \frac{dx}{x^2 + 4x + 5}$

SOLUCIÓN: $\boxed{I = \text{arc tg}\,(x+2) + C}$

187. Calcular: $I = \int \frac{4x - 5}{x^2 - 4x + 20}\,dx$

SOLUCIÓN: $\boxed{I = 2L\,|(x-2)^2 + 4^2| + \frac{3}{4}\,\text{arc tg}\,\frac{x-2}{4} + C}$

188. Calcular: $I = \int \frac{dx}{x\,(1+x^2)}$

SOLUCIÓN: $\boxed{I = L\left|\frac{x}{\sqrt{1+x^2}}\right| + C}$

189. Calcular: $I = \int \frac{dx}{3x^2 - 6x + 9}$

SOLUCIÓN: $\boxed{I = \frac{\sqrt{2}}{6}\,\text{arc tg}\,\frac{x-1}{\sqrt{2}} + C}$

190. Calcular: $I = \int \frac{4\,dx}{x^3 + 4x}$

SOLUCIÓN: $\boxed{I = L\left|\frac{x}{\sqrt{x^2+4}}\right| + C}$

191. Calcular: $I = \int \frac{4x^2 + 6}{x^3 + 3x}\,dx$

SOLUCIÓN: $\boxed{I = L\,|x^2\,(x^2+3)| + C}$

192. Calcular: $I = \int \frac{x^2 + x}{(x-1)\,(x^2+1)}\,dx$

SOLUCIÓN: $\boxed{I = L\,|x-1| + \text{arc tg}\,x + C}$

193. Calcular: $I = \int \frac{x - 18}{4x^3 + 9x}\,dx$

SOLUCIÓN: $\boxed{I = L\left|\frac{4x^2+9}{x^2}\right| + \frac{1}{6}\,\text{arc tg}\,\frac{2x}{3} + C}$

194. Calcular: $I = \int \frac{x^3 + 1}{(x-1)^4}\,dx$

SOLUCIÓN: $$\boxed{I = -\frac{2}{3(x-1)^3} - \frac{3}{2(x-1)^2} - \frac{3}{x-1} + L|x-1| + C}$$

195. Calcular: $I = \int \frac{dx}{x^4 - 13x^2 + 36}$

SOLUCIÓN: $$\boxed{I = \frac{1}{30} L\left|\frac{x-3}{x+3}\right| + \frac{1}{20} L\left|\frac{x+2}{x-2}\right| + C}$$

196. Calcular: $I = \int \frac{4x^2 + x + 1}{x^3 - 1}\,dx$

SOLUCIÓN: $$\boxed{I = L|(x-1)^2(x^2 + x + 1)| + C}$$

197. Calcular: $I = \int \frac{dx}{x^3 + 1}$

SOLUCIÓN: $$\boxed{I = \frac{1}{3} L|x+1| - \frac{1}{6} L|x^2 - x + 1| + \frac{\sqrt{3}}{3} \operatorname{arc\,tg} \frac{2x-1}{\sqrt{3}} + C}$$

198. Calcular: $I = \int \frac{dx}{x^3 + 8}$

SOLUCIÓN: $$\boxed{I = \frac{1}{12} L|x+2| - \frac{1}{24} L|x^2 - 2x + 4| + \frac{\sqrt{3}}{12} \operatorname{arc\,tg} \frac{x-1}{\sqrt{3}} + C}$$

199. Calcular: $I = \int \frac{dx}{\operatorname{sen} x}$

SOLUCIÓN: $$\boxed{I = L\left|\operatorname{tg} \frac{x}{2}\right| + C}$$

200. Calcular: $I = \int \frac{dx}{\cos x}$

SOLUCIÓN: $$\boxed{I = L\left|\frac{1 + \operatorname{tg} \frac{x}{2}}{1 - \operatorname{tg} \frac{x}{2}}\right| + C}$$

201. Calcular: $I = \int \frac{dx}{1 + \operatorname{sen} x + \cos x}$

SOLUCIÓN: $$\boxed{I = L\left|1 + \operatorname{tg} \frac{x}{2}\right| + C}$$

202. Calcular: $I = \int \frac{dx}{5 + 4\cos x}$

SOLUCIÓN: $$\boxed{I = \frac{2}{3} \operatorname{arc\,tg} \frac{\operatorname{tg} \frac{x}{2}}{3} + C}$$

203. Calcular: $I = \int \frac{dx}{\operatorname{sen}^3 x}$

SOLUCIÓN: $$\boxed{I = -\frac{1}{8\operatorname{tg}^2 \frac{x}{2}} + \frac{1}{2} L\left|\operatorname{tg} \frac{x}{2}\right| + \frac{\operatorname{tg}^2 \frac{x}{2}}{8} + C}$$

204. Calcular: $I = \int \frac{dx}{\operatorname{sen} x + \operatorname{ctg} x}$

SOLUCIÓN: $\boxed{I = \frac{1}{2} L \left| \operatorname{arc\,tg} \frac{x}{2} \right| - \frac{1}{4} \operatorname{arc\,tg}^2 \frac{x}{2} + C}$

205. Calcular: $I = \int \operatorname{sen}^3 x \, dx$

SOLUCIÓN: $\boxed{I = -\cos x + \frac{\cos^3 x}{3} + C}$

206. Calcular: $I = \int \cos^3 x \, dx$

SOLUCIÓN: $\boxed{I = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + C}$

207. Calcular: $I = \int \operatorname{sen}^5 x \, dx$

SOLUCIÓN: $\boxed{I = -\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + C}$

208. Calcular: $I = \int \cos^5 x \, dx$

SOLUCIÓN: $\boxed{I = \operatorname{sen} x - \frac{2\operatorname{sen}^3 x}{3} + \frac{\operatorname{sen}^5 x}{5} + C}$

209. Calcular: $I = \int \operatorname{sen}^4 x \cdot \cos x \, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{sen}^5 x}{5} + C}$

210. Calcular: $I = \int \cos^2 x \cdot \operatorname{sen}^3 x \, dx$

SOLUCIÓN: $\boxed{I = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C}$

211. Calcular: $I = \int \frac{\operatorname{sen}^3 x}{\cos^2 x} dx$

SOLUCIÓN: $\boxed{I = \frac{1}{\cos x} + \cos x + C}$

212. Calcular: $I = \int \frac{\cos^3 x}{\operatorname{sen}^2 x} dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{\operatorname{sen} x} - \operatorname{sen} x + C}$

213. Calcular: $I = \int \sqrt{\operatorname{sen} x} \cdot \cos^3 x \, dx$

SOLUCIÓN: $\boxed{I = \frac{2\operatorname{sen}^{3/2} x}{3} - \frac{2\operatorname{sen}^{7/2} x}{7} + C}$

214. Calcular: $I = \int \frac{\operatorname{sen}^3 x}{\cos x} dx$

SOLUCIÓN: $\boxed{I = -L|\cos x| + \frac{\cos^2 x}{2} + C}$

215. Calcular: $I = \int \frac{\cos x}{\operatorname{sen}^3 x} dx$

SOLUCIÓN: $\boxed{I = -\frac{1}{2\operatorname{sen}^2 x} + C}$

216. Calcular: $I = \int \frac{dx}{\cos^4 x}$

SOLUCIÓN: $\boxed{I = \operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} + C}$

217. Calcular: $I = \int \frac{dx}{\operatorname{sen}^4 x}$

SOLUCIÓN: $\boxed{I = -\operatorname{ctg} x - \frac{\operatorname{ctg}^3 x}{3} + C}$

218. Calcular: $I = \int \operatorname{sen}^3 x \cdot \cos^3 x \, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{sen}^4 x}{4} - \frac{\operatorname{sen}^6 x}{6} + C}$

219. Calcular: $I = \int \cos^4 x \, dx$

SOLUCIÓN: $\boxed{I = \frac{x}{4} + \frac{\operatorname{sen} 2x}{8} + \frac{x}{8} + \frac{\operatorname{sen} 4x}{32} + C}$

220. Calcular: $I = \int \operatorname{tg}^3 x \, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{tg}^2 x}{2} + L\,|\cos x| + C}$

221. Calcular: $I = \int \operatorname{ctg}^3 \frac{x}{3} \, dx$

SOLUCIÓN: $\boxed{I = -\frac{3 \operatorname{ctg}^2 \frac{x}{3}}{2} - 3L\left|\operatorname{sen} \frac{x}{3}\right| + C}$

222. Calcular: $I = \int \sec^4 2x \, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{tg} 2x}{2} + \frac{3 \operatorname{tg}^3 2x}{2} + C}$

223. Calcular: $I = \int \operatorname{sen} 3x \cdot \operatorname{sen} 2x \, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{sen} x}{2} - \frac{\operatorname{sen} 5x}{10} + C}$

224. Calcular: $I = \int \operatorname{sen} 4x \cdot \cos 2x \, dx$

SOLUCIÓN: $\boxed{I = -\frac{\cos 6x}{12} - \frac{\cos 2x}{4} + C}$

225. Calcular: $I = \int \cos 4x \cdot \cos 3x \, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{sen} 7x}{14} + \frac{\operatorname{sen} x}{2} + C}$

226. Calcular: $I = \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

SOLUCIÓN: $\boxed{I = 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6L\,|1 + \sqrt[6]{x}| + C}$

227. Calcular: $I = \int \frac{dx}{\sqrt[4]{x^3} - \sqrt{x}}$

SOLUCIÓN: $\boxed{I = 4\sqrt[4]{x} + 4L\,|\sqrt[4]{x} - 1| + C}$

228. Calcular: $I = \int \frac{x^{1/4}}{1 + x^{1/2}} \, dx$

SOLUCIÓN: $\boxed{I = 4\left[\frac{\sqrt[4]{x}}{3} - \sqrt[4]{x} + \operatorname{arc\,tg} \sqrt[4]{x}\right] + C}$

229. Calcular: $I = \int \frac{\sqrt{x} - \sqrt[6]{x}}{\sqrt[3]{x} + 1} \, dx$

SOLUCIÓN: $\boxed{I = 6\left[\frac{\sqrt[6]{x^7}}{7} - \frac{2\sqrt[6]{x^5}}{5} + \frac{2\sqrt[6]{x^3}}{3} - 2\sqrt[6]{x} + 2 \operatorname{arc\,tg} \sqrt[6]{x}\right] + C}$

230. Calcular: $I = \int \frac{x^5}{\sqrt{x^3 - 1}}\, dx$

SOLUCIÓN: $\boxed{I = \frac{2}{3}\left[(\sqrt{x^3 - 1})^3 + \sqrt{x^3 - 1}\right] + C}$

231. Calcular: $I = \int \frac{2 + x}{\sqrt{x + 3}}\, dx$

SOLUCIÓN: $\boxed{I = \frac{2x\,(x + 3)^{1/2}}{3} + C}$

232. Calcular: $I = \int \frac{dx}{x\sqrt{x - 1}}$

SOLUCIÓN: $\boxed{I = 2 \operatorname{arc\,tg} \sqrt{x - 1} + C}$

233. Calcular: $I = \int \frac{x + 3}{(x + 5)\sqrt{x + 4}}\, dx$

SOLUCIÓN: $\boxed{I = 2\left[\sqrt{x + 4} - 2 \operatorname{arc\,tg} \sqrt{x + 4}\right] + C}$

234. Calcular: $I = \int \frac{dx}{\sqrt{1 + \sqrt{1 + x}}}$

SOLUCIÓN: $\boxed{I = \frac{\sqrt{1 + \sqrt{1 + x}}}{3}\left[4\sqrt{1 + x} - 8\right] + C}$

235. Calcular: $I = \int \sqrt{1 - x^2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{\operatorname{arc\,sen} x}{2} + \frac{1}{2}x\sqrt{1 - x^2} + C}$

236. Calcular: $I = \int \sqrt{4 - x^2}\, dx$

SOLUCIÓN: $\boxed{I = 2 \operatorname{arc\,sen} \frac{x}{2} + \frac{x}{2}\sqrt{4 - x^2} + C}$

237. Calcular: $I = \int \sqrt{25 - 9x^2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{25}{6} \operatorname{arc\,sen} \frac{3x}{5} + \frac{x}{2}\sqrt{25 - 9x^2} + C}$

238. Calcular: $I = \int \frac{dx}{\sqrt{25 - 9x^2}}$

SOLUCIÓN: $\boxed{I = \frac{1}{3} \operatorname{arc\,sen} \frac{3x}{5} + C}$

239. Calcular: $I = \int \frac{x}{\sqrt{x^2 - 4}}\, dx$

SOLUCIÓN: $\boxed{I = \sqrt{x^2 - 4} + C}$

240. Calcular: $I = \int \sqrt{1 + 9x^2}\, dx$

SOLUCIÓN: $\boxed{I = \frac{x}{2}\sqrt{1 + 9x^2} + \frac{1}{6} L\left|3x + \sqrt{1 + 9x^2}\right| + C}$

241. Calcular: $I = \int \frac{\sqrt{x^2 - 1}}{x}\, dx$

SOLUCIÓN: $\boxed{I = \sqrt{x^2 - 1} - \operatorname{arc\,sen} x + C}$

242. Calcular: $I = \int \sqrt{8x - x^2}\, dx$

SOLUCIÓN: $\boxed{I = 8 \operatorname{arc\,sen} \frac{x - 4}{4} + \frac{x - 4}{2} \cdot \sqrt{8x - x^2} + C}$

243. Calcular: $I = \int \frac{dx}{\sqrt{(x^2 + a^2)^3}}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{a^2\sqrt{x^2 + a^2}} + C}}$

244. Calcular: $I = \int \frac{x^2}{\sqrt{(1 - x^2)^3}}\, dx$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{\sqrt{1 - x^2}} - \text{arc sen } x + C}}$

245. Calcular: $I = \int \frac{dx}{\sqrt{x^2 + 1}\,(\sqrt{x^2 + 1} + x)}$

SOLUCIÓN: $\boxed{\mathbf{I = x - \sqrt{x^2 + 1} + C}}$

246. Calcular: $I = \int \sqrt{3 - 2x - x^2}\, dx$

SOLUCIÓN: $\boxed{\mathbf{I = 2\,\text{arc sen}\,\frac{x + 1}{2} + \frac{x - 1}{2}\cdot\sqrt{4 - (x + 1)^2} + C}}$

247. Calcular: $I = \int \frac{dx}{\sqrt{4ax - x^2}}$

SOLUCIÓN: $\boxed{\mathbf{I = \text{arc sen}\,\frac{x - a}{2a} + C}}$

248. Calcular: $I = \int \frac{x}{4 - x^2 + \sqrt{4 - x^2}}\, dx$

SOLUCIÓN: $\boxed{\mathbf{I = -L\left|\frac{\sqrt{4 - x^2} + 1}{2}\right| + C}}$

RESOLUCIONES

1. RESOLUCIÓN

$$I = \int x^3\, dx = \frac{x^4}{4} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^4}{4} + C}}$

2. RESOLUCIÓN

$$I = \int \frac{dx}{x^3} = \int x^{-3}\, dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{1}{2x^2} + C}}$

3. RESOLUCIÓN

$$I = \int x\sqrt[3]{x}\, dx = \int x \cdot x^{1/3}\, dx = \int x^{4/3}\, dx = \frac{x^{7/3}}{\frac{7}{3}} + C =$$

$$= \frac{3}{7}x^{7/3} + C = \frac{3}{7}\sqrt[3]{x^7} + C = \frac{3}{7}x^2\sqrt[3]{x} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{3}{7}x^2 \cdot \sqrt[3]{x} + C}}$

4. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt[3]{x}} = \int \frac{dx}{x^{1/3}} \int x^{-1/3}\, dx = \frac{x^{2/3}}{\frac{2}{3}} + C = \frac{3}{2}\sqrt[3]{x^2} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{3}{2}\cdot\sqrt[3]{x^2} + C}}$

5. RESOLUCIÓN

$$I = \int \left(\frac{2}{3} x^4 - \frac{2}{4} x^2 + 1 \right) dx = \frac{2}{3} \int x^4 \, dx - \frac{1}{2} \int x^2 \, dx + \int dx =$$

$$= \frac{2}{3} \cdot \frac{x^5}{5} - \frac{1}{2} \cdot \frac{x^3}{3} + x + C = \frac{2x^5}{15} - \frac{x^3}{6} + x + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2x^5}{15} - \frac{x^3}{6} + x + C}}$

6. RESOLUCIÓN

$$I = \int \left(\frac{2}{3} x^{3/2} - \frac{3}{4} x^{1/2} - \frac{2}{3} \right) dx = \frac{2}{3} \int x^{3/2} \, dx -$$

$$- \frac{3}{4} \int x^{1/2} \, dx - \frac{2}{3} \int dx = \frac{2}{3} \cdot \frac{x^{5/3}}{\frac{5}{3}} - \frac{3}{4} \cdot \frac{x^{3/2}}{\frac{3}{2}} - \frac{2}{3} x +$$

$$+ C = \frac{2x^{5/3}}{5} - \frac{x^{3/2}}{2} - \frac{2x}{3} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2x^{5/3}}{5} - \frac{x^{3/2}}{2} - \frac{2x}{3} + C}}$

7. RESOLUCIÓN

$$I = \int \left(- \frac{3}{x^4} - \frac{1}{x^3} + \frac{2}{x^5} \right) dx = - 3 \int x^{-4} \, dx - \int x^{-3} \, dx +$$

$$+ 2 \int x^{-5} \, dx = - 3 \cdot \frac{x^{-3}}{-3} - \frac{x^{-2}}{-2} + 2 \cdot \frac{x^{-4}}{-4} + C =$$

$$= \frac{1}{x^3} + \frac{1}{2x^2} - \frac{1}{2x^4} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{x^3} + \frac{1}{2x^2} - \frac{1}{2x^4} + C}}$

8. RESOLUCIÓN

$$I = \int (x^2 - 3x + 4) \, dx = \int x^2 \, dx - 3 \int x \, dx + 4 \int dx =$$

$$= \frac{x^3}{3} - 3 \cdot \frac{x^2}{2} + 4x + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^3}{3} - \frac{3x^2}{2} + 4x + C}}$

9. RESOLUCIÓN

$$I = \int \frac{2x^3 + 2x + 1}{1 + x^2} = \int \left(2x + \frac{1}{1 + x^2} \right) dx =$$

$$= 2 \int x \, dx + \int \frac{dx}{1 + x^2} = 2 \frac{x^2}{2} + \operatorname{arc\,tg} x + C =$$

$$= x^2 + \operatorname{arc\,tg} x + C$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} 2x^3 + 2x + 1 & x^2 + 1 \\ \cline{2-2} -2x^3 - 2x & 2x \\ \cline{1-1} + 1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = x^2 + \operatorname{arc\,tg} x + C}}$

10. RESOLUCIÓN

$$I = \int \frac{(x - 1) \, dx}{x + 1} = \int \frac{x - 1 + 1 - 1}{x + 1} dx = \int \frac{x + 1 - 2}{x + 1} dx =$$

$$= \int \left(1 - \frac{2}{x + 1} \right) dx = \int dx - 2 \int \frac{dx}{x + 1} = x - 2L \, (x + 1) + C$$

SOLUCIÓN: $\boxed{\mathbf{I = x - 2 L \, |x + 1| + C}}$

11. RESOLUCIÓN

$$I = \int \frac{x^4 - 5x^2 + 10}{x^2}\,dx = \int \left(x^2 - 5 + \frac{10}{x^2}\right)dx = \int x^2\,dx -$$

$$- 5\int dx + 10\int \frac{dx}{x^2} = \frac{x^3}{3} - 5x + 10\int x^{-2}\,dx =$$

$$= \frac{x^3}{3} - 5x - \frac{10}{x} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^3}{3} - 5x - \frac{10}{x} + C}}$

12. RESOLUCIÓN

$$I = \int (\sqrt{a} - \sqrt{x})^2\,dx = \int (a - 2\sqrt{a}\sqrt{x} + x)\,dx = a\int dx -$$

$$- 2\sqrt{a}\int x^{1/2}\,dx + \int x\,dx = ax - 2\sqrt{a}\cdot \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^2}{2} + C =$$

$$= ax - \frac{4x\sqrt{ax}}{3} + \frac{x^2}{2} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = ax - \frac{4x\sqrt{ax}}{3} + \frac{x^2}{2} + C}}$

13. RESOLUCIÓN

$$I = \int \sqrt{x}\,(\sqrt{a} - \sqrt{x})^2\,dx = \int \sqrt{x}\,(a - 2\sqrt{a}\sqrt{x} + x)\,dx =$$

$$= a\int \sqrt{x}\,dx - 2\sqrt{a}\int x\,dx + \int x\sqrt{x}\,dx = a\int x^{1/2}dx - 2\sqrt{a}\int x\,dx +$$

$$+ \int x^{3/2}\,dx = a\cdot \frac{x^{3/2}}{\frac{3}{2}} - 2\sqrt{a}\cdot \frac{x^2}{2} + \frac{x^{5/2}}{\frac{5}{2}} + C =$$

$$= \frac{2ax^{3/2}}{3} - x^2\sqrt{a} + \frac{2x^{5/2}}{5} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2ax^{3/2}}{3} - x^2\sqrt{a} + \frac{2x^{5/2}}{5} + C}}$

14. RESOLUCIÓN

$$I = \int 10^x\,dx = \frac{10^x}{L\,10} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{10^x}{L\,10} + C}}$

15. RESOLUCIÓN

$$I = \int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx = \int \left(x + 2 + \frac{1}{x}\right)dx =$$

$$= \int x\,dx + 2\int dx + \int \frac{dx}{x} = \frac{x^2}{2} + 2x + L\,|x| + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^2}{2} + 2x + L\,|x| + C}}$

16. RESOLUCIÓN

$$I = \int \frac{dx}{sen^2 x\cdot cos^2 x} = \int \frac{sen^2 x + cos^2 x}{sen^2 x\cdot cos^2 x}\,dx =$$

$$= \int \frac{sen^2 x\,dx}{sen^2 x\cdot cos^2 x} + \int \frac{cos^2 x\,dx}{sen^2 x\cdot cos^2 x} = \int \frac{dx}{cos^2 x} +$$

$$+ \int \frac{dx}{sen^2 x} = tg\,x - ctg\,x + C$$

CÁLCULOS AUXILIARES

$sen^2 x + cos^2 x = 1$

SOLUCIÓN: $\boxed{\mathbf{I = tg\,x - ctg\,x + C}}$

17. RESOLUCIÓN

$$I = \int tg^2 x\,dx = \int (tg^2 x + 1 - 1)\,dx = \int (tg^2 x + 1)\,dx - \int dx =$$

$$= \int sec^2 x\,dx - \int dx = tg\,x - x + C$$

CÁLCULOS AUXILIARES

$$1 + tg^2 x = sec^2 x = \frac{1}{\cos^2 x}$$

SOLUCIÓN: $\boxed{\mathbf{I = tg\ x - x + C}}$

18. RESOLUCIÓN

$$I = \int \frac{3\cos x + 2 - 2\,sen^2 x}{\cos x}\,dx = \int \frac{3\cos x + 2\,(1 - sen^2 x)\,dx}{\cos x} =$$

$$= \int \frac{3\cos x + 2\cos^2 x}{\cos x}\,dx = 3\int dx + 2\int \cos x\,dx =$$

$$= 3x + 2\,sen\,x + C$$

CÁLCULOS AUXILIARES

$$1 - sen^2 x = \cos^2 x$$

SOLUCIÓN: $\boxed{\mathbf{I = 3x + 2\ sen\ x + C}}$

19. RESOLUCIÓN

$$I = \int \frac{(\sqrt{a} + \sqrt{x})^2}{\sqrt{x}}\,dx = \int \frac{(a + 2\sqrt{a}\cdot\sqrt{x} + x)\,dx}{\sqrt{x}} =$$

$$= a\int \frac{dx}{\sqrt{x}} + 2\sqrt{a}\int dx + \int \frac{x}{\sqrt{x}}\,dx = a\cdot 2\sqrt{x} + 2\sqrt{a}\cdot x +$$

$$+ \int x^{1/2}\,dx = 2a\sqrt{x} + 2x\sqrt{a} + \frac{2x^{3/2}}{3} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = 2a\sqrt{x} + 2x\sqrt{a} + \frac{2}{3}x\sqrt{x} + C}}$

20. RESOLUCIÓN

$$I = \int \frac{x^3 - 6x + 5}{x - 2}\,dx = \int \left(x^2 + 2x - 2 + \frac{1}{x - 2}\right)dx =$$

$$= \int x^2\,dx + 2\int x\,dx - 2\int dx + \int \frac{dx}{x - 2} =$$

$$= \frac{x^3}{3} + 2\cdot\frac{x^2}{2} - 2x + L\,|x - 2| + C =$$

$$= \frac{x^3}{3} + x^2 - 2x + L\,|x - 2| + C$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} x^3 - 6x + 5 & x - 2 \\ \hline -x^3 + 2x^2 & x^2 + 2x - 2 \\ \hline 2x^2 - 6x & \\ -2x^2 + 4x & \\ \hline -2x + 5 & \\ 2x - 4 & \\ \hline 1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^3}{3} + x^2 - 2x + L\,|x - 2| + C}}$

21. RESOLUCIÓN

$$I = \int \left(\frac{sen\ 2x}{sen\ x\cos x} + \cos x\right)dx = \int \frac{sen\ 2x}{sen\ x\cos x}\,dx +$$

$$+ \int \cos x\,dx = \int \frac{2\,sen\,x\cos x}{sen\,x\cos x}\,dx + \int \cos x\,dx =$$

$$= 2\int dx + \int \cos x\,dx = 2x + sen\,x + C$$

CÁLCULOS AUXILIARES

$$sen\ 2x = 2\,sen\,x\cos x$$

SOLUCIÓN: $\boxed{\mathbf{I = 2x + sen\ x + C}}$

22. RESOLUCIÓN

$$I = \int (4x - 2)^5\,dx = \int t^5 \cdot \frac{dt}{4} = \frac{1}{4}\int t^5\,dt = \frac{1}{4}\cdot\frac{t^6}{6} + C =$$

$$= \frac{(4x - 2)^6}{24} + C$$

CÁLCULOS AUXILIARES

$4x - 2 = t$

$4\,dx = dt$

$dx = \dfrac{dt}{4}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(4x - 2)^6}{24} + C}}$

23. RESOLUCIÓN

$$I = \int x\,(3x^2 + 1)\,dx = \int t \cdot \frac{dt}{6} = \frac{1}{6}\int t\,dt = \frac{1}{6} \cdot \frac{t^2}{2} + C =$$

$$= \frac{(3x^2 + 1)^2}{12} + C$$

CÁLCULOS AUXILIARES

$3x^2 + 1 = t$

$6x\,dx = dt$

$x\,dx = \dfrac{dt}{6}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(3x^2 + 1)^2}{12} + C}}$

24. RESOLUCIÓN

$$I = \int \frac{2x + 1}{x^2 + x - 3}\,dx = \int \frac{dt}{t} = L\,|t| + C = L\,|x^2 + x - 3| + C$$

CÁLCULOS AUXILIARES

$x^2 + x - 3 = t$

$(2x + 1)\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|x^2 + x - 3| + C}}$

25. RESOLUCIÓN

$$I = \int (x^3 - 5x^2 + 4x)\,(3x^2 - 10x + 4)\,dx =$$

$$= \int t\,dt = \frac{t^2}{2} + C = \frac{(x^3 - 5x^2 + 4x)^2}{2} + C$$

CÁLCULOS AUXILIARES

$x^3 - 5x^2 + 4x = t$

$(3x^2 - 10x + 4)\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(x^3 - 5x^2 + 4x)^2}{2} + C}}$

26. RESOLUCIÓN

$$I = \int 2x\sqrt{1 + 3x^2}\,dx = \int \sqrt{t} \cdot \frac{dt}{3} = \frac{1}{3}\int t^{1/2}\,dt =$$

$$= \frac{1}{3} \cdot \frac{t^{3/2}}{\frac{3}{2}} + C = \frac{2}{9}\sqrt{t^3} + C = \frac{2}{9}\sqrt{(1 + 3x^2)^3} + C =$$

$$= \frac{2}{9}\,(1 + 3x^2)\,\sqrt{(1 + 3x^2)} + C$$

CÁLCULOS AUXILIARES

$1 + 3x^2 = t$

$3 \cdot 2x\,dx = dt$

$2x\,dx = \dfrac{dt}{3}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2}{9}\,(1 + 3x^2)\,\sqrt{(1 + 3x^2)} + C}}$

27. RESOLUCIÓN

$$I = \int \frac{2x\,dx}{\sqrt{8 + x^2}} = \int \frac{2t\,dt}{t} = 2\int dt = 2t + C = 2\sqrt{8 + x^2} + C$$

CÁLCULOS AUXILIARES

$\sqrt{8 + x^2} = t$

$8 + x^2 = t^2$

$2x\,dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{8 + x^2} + C}}$

28. RESOLUCIÓN

$$I = \int (x + 3)(x^2 + 6x - 4)\,dx = \int t \cdot \frac{dt}{2} = \frac{1}{2}\int t\,dt =$$

$$= \frac{1}{2} \cdot \frac{t^2}{2} + C = \frac{(x^2 + 6x - 4)^2}{4} + C$$

CÁLCULOS AUXILIARES

$x^2 + 6x - 4 = t$

$(2x + 6)\,dx = dt$

$2(x + 3)\,dx = dt$

$(x + 3)\,dx = \dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(x^2 + 6x - 4)^2}{4} + C}}$

29. RESOLUCIÓN

$$I = \int (x + 3)\,sen\,(x^2 + 6x - 4)\,dx = \int sen\,t \cdot \frac{dt}{2} =$$

$$= \frac{1}{2}\int sen\,t\,dt = -\frac{1}{2}\cos t + C = -\frac{1}{2}\cos(x^2 + 6x - 4) + C$$

CÁLCULOS AUXILIARES

$x^2 + 6x - 4 = t$

$(2x + 6)\,dx = dt$

$(x + 3)\,dx = \dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{2}\cos\,(x^2 + 6x - 4) + C}}$

30. RESOLUCIÓN

$$I = \int x\sqrt{x - 1}\,dx = \int (t^2 + 1)\,t \cdot 2t\,dt = 2\int (t^4 + t^2)\,dt =$$

$$= 2\int t^4\,dt + 2\int t^2\,dt = 2 \cdot \frac{t^5}{5} + 2 \cdot \frac{t^3}{3} + C =$$

$$= \frac{2(x - 1)^{5/2}}{5} + \frac{2(x - 1)^{3/2}}{3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x - 1} = t$

$x - 1 = t^2 \Rightarrow x = t^2 + 1$

$dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2(x - 1)^{5/2}}{5} + \dfrac{2(x - 1)^{3/2}}{3} + C}}$

31. RESOLUCIÓN

$$I = \int x\,sen\,x^2\,dx = \frac{1}{2}\int sen\,t\,dt = -\frac{1}{2}\cos t + C =$$

$$= -\frac{1}{2}\cos x^2 + C$$

CÁLCULOS AUXILIRES

$x^2 = t$

$2x\,dx = dt$

$x\,dx = \dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{\cos x^2}{2} + C}}$

32. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{x}\cos^2\sqrt{x}} = \int \frac{2\,dt}{\cos^2 t} = 2\,tg\,t + C = 2\,tg\,\sqrt{x} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x} = t$

$\dfrac{dx}{2\sqrt{x}} = dt$

$\dfrac{dx}{\sqrt{x}} = 2\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\,tg\,\sqrt{x} + C}}$

33. RESOLUCIÓN

$$I = \int \frac{x\,dx}{(x+1)(x-1)} = \int \frac{x\,dx}{x^2-1} = \frac{1}{2}\int \frac{dt}{t} =$$

$$= \frac{1}{2} L\,|t| + C = \frac{1}{2} L\,|x^2 - 1| + C$$

CÁLCULOS AUXILIARES

$x^2 - 1 = t$

$2x\,dx = dt$

$x\,dx = \dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{L\,|x^2 - 1|}{2} + C}}$

34. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{7x-2}} = \int \frac{\frac{2}{7}\,t\,dt}{t} = \frac{2}{7}\int dt = \frac{2}{7}\,t + C =$$

$$= \frac{2}{7}\sqrt{7x-2} + C$$

CÁLCULOS AUXILIARES

$7x - 2 = t^2$

$7\,dx = 2t\,dt$

$dx = \dfrac{2}{7}\,t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2\sqrt{7x-2}}{7} + C}}$

35. RESOLUCIÓN

$$I = \int \frac{arc\ sen\ x}{\sqrt{1-x^2}}\,dx = \int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}(arc\ sen\ x)^2 + C$$

CÁLCULOS AUXILIARES

$arc\ sen\ x = t$

$\dfrac{dx}{\sqrt{1-x^2}} = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(arc\ sen\ x)^2}{2} + C}}$

36. RESOLUCIÓN

$$I = \int x\sqrt{5x^2+1}\,dx = \frac{1}{10}\int \sqrt{t}\,dt = \frac{1}{10}\int t^{1/2}\,dt =$$

$$= \frac{1}{10} \cdot \frac{t^{3/2}}{\frac{3}{2}} + C = \frac{1}{15}(5x^2+1)^{3/2} + C$$

CÁLCULOS AUXILIARES

$5x^2 + 1 = t$

$10x\,dx = dt$

$x\,dx = \dfrac{dt}{10}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(5x^2+1)^{3/2}}{15} + C}}$

37. RESOLUCIÓN

$$I = \int \frac{L\,x}{x}\,dx = \int t\,dt = \frac{t^2}{2} + C = \frac{(L\,x)^2}{2} + C$$

CÁLCULOS AUXILIARES

$Lx = t$

$\dfrac{dx}{x} = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(L\,x)^2}{2} + C}}$

38. RESOLUCIÓN

$$I = \int \frac{dx}{(x-1)^2}$$

$$I = \int \frac{dt}{t^2} = \int t^{-2}\,dt = \frac{t^{-1}}{-1} + C = -\frac{1}{t} + C = -\frac{1}{(x-1)} + C$$

CÁLCULOS AUXILIARES

$x - 1 = t$

$dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{x-1} + C}}$

39. RESOLUCIÓN

$$I = \int \frac{(L\,x)^3}{x}\,dx = \int t^3\,dt = \frac{t^4}{4} + C = \frac{(L\,x)^4}{4} + C$$

CÁLCULOS AUXILIARES

$Lx = t$

$\dfrac{dx}{x} = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(L\,x)^4}{4} + C}}$

40. RESOLUCIÓN

$$I = \int \frac{dx}{(1+x)\sqrt{x}} = \int \frac{2t\,dt}{(1+t^2)\,t} = 2\int \frac{dt}{1+t^2} =$$

$$= \text{arc tg}\, t + C = 2\,\text{arc tg}\,\sqrt{x} + C$$

CÁLCULOS AUXILIARES

$x = t^2 \Rightarrow t = \sqrt{x}$

$dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\,arc\,tg\,\sqrt{x} + C}}$

41. RESOLUCIÓN

$$I = \int \frac{x^2\,dx}{\sqrt{5x^3+7}} = \frac{2}{15}\int \frac{t\,dt}{t} = \frac{2}{15}\int dt =$$

$$= \frac{2}{15}\,t + C = \frac{2}{15}\sqrt{5x^3+7} + C$$

CÁLCULOS AUXILIARES

$\sqrt{5x^3+7} = t$

$5x^3 + 7 = t^2$

$15x^2\,dx = 2t\,dt$

$x^2\,dx = \dfrac{2t\,dt}{15}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2}{15}\sqrt{5x^3+7} + C}}$

42. RESOLUCIÓN

$$I = \int \frac{(6x^3 - 11x^2 - 19x - 7)}{3x+2}\,dx = \int \Big(2x^2 - 5x - 3 -$$

$$- \frac{1}{3x+2}\Big)dx = 2\int x^2\,dx - 5\int x\,dx - 3\int dx - \int \frac{dx}{3x+2} =$$

$$= 2\cdot\frac{x^3}{3} - 5\cdot\frac{x^2}{2} - 3x - \frac{1}{3}\int \frac{3\,dx}{3x+2} =$$

$$= \frac{2x^3}{3} - \frac{5x^2}{2} - 3x - \frac{1}{3}\,L\,|3x+2| + C$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} 6x^3 - 11x^2 - 19x - 7 & 3x + 2 \\ \cline{2-2} -6x^3 - 4x^2 & 2x^2 - 5x - 3 \\ \cline{1-1} -15x^2 - 19x & \\ 15x^2 + 10x & \\ \cline{1-1} -9x - 7 & \\ 9x + 6 & \\ \cline{1-1} -1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2x^3}{3} - \frac{5x^2}{2} - 3x - \frac{1}{3} L\,|3x + 2| + C}}$

43. RESOLUCIÓN

$$I = \int \frac{\sqrt{x}}{\sqrt{x} - 2}\,dx = \int \frac{t}{t - 2} \cdot 2t\,dt = 2\int \frac{t^2\,dt}{t - 2} =$$

$$= 2\int \left(t + 2 + \frac{4}{t - 2} \right) dt = 2\int t\,dt + 4\int dt + 8\int \frac{dt}{t - 2} =$$

$$= t^2 + 4t + 8\,L\,|t - 2| + C = x + 4\sqrt{x} + 8\,L\,|\sqrt{x} - 2| + C$$

CÁLCULOS AUXILIARES

$\sqrt{x} = t \quad ; \quad x = t^2$

$dx = 2t\,dt$

$$\begin{array}{r|l} t^2 & t - 2 \\ \cline{2-2} -t^2 + 2t & t + 2 \\ \cline{1-1} 2t & \\ -2t + 4 & \\ \cline{1-1} 4 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = x + 4\sqrt{x} + 8\,L\,|\sqrt{x} - 2| + C}}$

44. RESOLUCIÓN

$$I = \int \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}\,dx = \int \frac{t}{1 + t} \cdot 3t^2\,dt = 3\int \frac{t^3}{t + 1}\,dt =$$

$$= 3\int \left(t^2 - t + 1 - \frac{1}{t + 1} \right) dt =$$

$$= 3 \cdot \frac{t^3}{3} - 3 \cdot \frac{t^2}{2} + 3t - 3\,L\,|t + 1| + C =$$

$$= x - \frac{3}{2}\sqrt[3]{x^2} + 3\sqrt[3]{x} - 3\,L\,|\sqrt[3]{x} + 1| + C$$

CÁLCULOS AUXILIARES

$\sqrt[3]{x} = t \quad ; \quad x = t^3$

$dx = 3t^2\,dt$

$$\begin{array}{r|l} t^3 & t + 1 \\ \cline{2-2} -t^3 - t^2 & t^2 - t + 1 \\ \cline{1-1} -t^2 & \\ t^2 + t & \\ \cline{1-1} t & \\ -t - 1 & \\ \cline{1-1} -1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = x - \frac{3}{2}\sqrt[3]{x^2} + 3\sqrt[3]{x} - 3\,L\,|\sqrt[3]{x} + 1| + C}}$

45. RESOLUCIÓN

$$I = \int (e^x - 3e^{2x} + 4e^{3x})\,dx = \int e^x\,dx - 3\int e^{2x}\,dx +$$

$$+ 4\int e^{3x}\,dx = e^x - 3 \cdot \frac{e^{2x}}{2} + 4 \cdot \frac{e^{3x}}{3} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = e^x - \frac{3}{2}e^{2x} + \frac{4}{3}e^{3x} + C}}$

46. RESOLUCIÓN

$$I = \int sen^2 3x \cdot \cos 3x\,dx = \int t^3 \cdot \frac{dt}{3} = \frac{1}{3}\int t^3\,dt =$$

$$= \frac{1}{3} \cdot \frac{t^4}{4} + C = \frac{sen^4 3x}{12} + C$$

CÁLCULOS AUXILIARES

$sen\ 3x = t$

$3\ cos\ 3x\ dx = dt$

$cos\ 3x\ dx = \dfrac{dt}{3}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{sen^4\ 3x}{12} + C}}$

47. RESOLUCIÓN

$$I = \int \frac{e^x\,dx}{e^x + 2} = \int \frac{dt}{t} = L\,|t| + C = L\,|e^x + 2| + C$$

CÁLCULOS AUXILIARES

$e^x + 2 = t$

$e^x\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|e^x + 2| + C}}$

48. RESOLUCIÓN

$$I = \int \frac{2x\,dx}{1 + \sqrt{x}} = 2\int \frac{x\,dx}{1 + \sqrt{x}} = 2\int \frac{t^2\,2t\,dt}{1 + t} = 4\int \frac{t^3}{t + 1}\,dt =$$

$$= 4\int \left(t^2 - t + 1 - \frac{1}{t + 1}\right)dt = 4 \cdot \frac{t^3}{3} - 4 \cdot \frac{t^2}{2} + 4t -$$

$$- 4\,L\,|t + 1| + C = \frac{4}{3}\sqrt{x^3} - 2x + 4\sqrt{x} - 4\,L\,|\sqrt{x} + 1| + C =$$

$$= \frac{4}{3}x\sqrt{x} - 2x + 4\sqrt{x} - 4\,L\,|\sqrt{x} + 1| + C$$

CÁLCULOS AUXILIARES

$\sqrt{x} = t \quad ; \quad x = t^2$

$dx = 2t\,dt$

$$\begin{array}{r|l} t^3 & t + 1 \\ \hline -t^3 - t^2 & t^2 - t + 1 \\ \hline -t^2 & \\ t^2 + t & \\ \hline t & \\ -t - 1 & \\ \hline -1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{4}{3}\,x\sqrt{x} - 2x + 4\sqrt{x} - 4\,L\,|\sqrt{x} + 1| + C}}$

49. RESOLUCIÓN

$$I = \int \frac{ctg\ x}{\sqrt{sen\ x}}\,dx = \int \frac{\frac{cos\ x}{sen\ x}\,dx}{\sqrt{sen\ x}} = \int \frac{cos\ x\,dx}{sen\ x\sqrt{sen\ x}} =$$

$$= \int \frac{2t\,dt}{t^2 \cdot t} = 2\int \frac{dt}{t^2} = 2 \cdot \frac{t^{-1}}{-1} + C = -\frac{2}{t} + C =$$

$$= -\frac{2}{\sqrt{sen\ x}} + C$$

CÁLCULOS AUXILIARES

$\sqrt{sen\ x} = t \quad ; \quad sen\ x = t^2$

$cos\ x\,dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{2}{\sqrt{sen\ x}} + C}}$

50. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{x}} = \int \frac{2t\,dt}{t} = 2\int dt = 2t + C = 2\sqrt{x} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x} = t \quad ; \quad x = t^2$

$dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{x} + C}}$

51. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt[3]{x}} = \int \frac{3t^2\,dt}{t} = 3\int t\,dt = 3 \cdot \frac{t^2}{2} + C = \frac{3\sqrt[3]{x^2}}{2} + C$$

CÁLCULOS AUXILIARES

$\sqrt[3]{x} = t \quad ; \quad x = t^3$

$dx = 3t^2\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{3\sqrt[3]{x^2}}{2} + C}}$

52. RESOLUCIÓN

$$I = \int \sqrt{ax}\,dx = \frac{2}{a}\int t^2\,dt = \frac{2}{a} \cdot \frac{t^3}{3} + C =$$

$$= \frac{2}{a} \cdot \frac{ax\sqrt{ax}}{3} + C = \frac{2x\sqrt{ax}}{3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{ax} = t \quad ; \quad ax = t^2$

$a\,dx = 2t\,dt$

$dx = \dfrac{2t\,dt}{a}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2x\sqrt{ax}}{3} + C}}$

53. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{a - bx}} = -\frac{2}{b}\int \frac{t\,dt}{t} = -\frac{2}{b}\int dt = -\frac{2}{b}t + C =$$

$$= -\frac{2}{b}\sqrt{a - bx} + C$$

CÁLCULOS AUXILIARES

$\sqrt{a - bx} = t \quad ; \quad a - bx = t^2$

$-b\,dx = 2t\,dt$

$dx = -\dfrac{2t\,dt}{b}$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{2\sqrt{a - bx}}{b} + C}}$

54. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{9 - 16x^2}} = \int \frac{\frac{3}{4}\,dt}{\sqrt{9 - 16 \cdot \frac{9}{16}t^2}} = \frac{3}{4}\int \frac{dt}{\sqrt{9 - 9t^2}} =$$

$$= \frac{1}{4}\,\text{arc sen}\,t + C = \frac{1}{4}\,\text{arc sen}\,\frac{4x}{3} + C$$

CÁLCULOS AUXILIARES

$x = \dfrac{3}{4}t \quad ; \quad t = \dfrac{4x}{3}$

$dx = \dfrac{3}{4}\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{4}\,arc\,sen\,\frac{4x}{3} + C}}$

55. RESOLUCIÓN

$$I = \int \sqrt{a + bx}\, dx = \frac{2}{b}\int t^2\, dt = \frac{2}{b}\cdot\frac{t^3}{3} + C =$$

$$= \frac{2}{b}\cdot\frac{(\sqrt{a + bx})^3}{3} + C = \frac{2\,(a + bx)^{3/2}}{3b} + C$$

CÁLCULOS AUXILIARES

$\sqrt{a + bx} = t$

$a + bx = t^2$

$b\,dx = 2t\,dt$

$$dx = \frac{2t\,dt}{b}$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2(a + bx)^{3/2}}{3b} + C}}$

56. RESOLUCIÓN

$$I = \int x\,(2 + x^2)^2\, dx = \int t^2 \cdot \frac{dt}{2} = \frac{1}{2}\int t^2\, dt = \frac{1}{2}\cdot\frac{t^3}{3} + C =$$

$$= \frac{(2 + x^2)^3}{6} + C$$

CÁLCULOS AUXILIARES

$2 + x^2 = t$

$2x\,dx = dt$

$$x\,dx = \frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(2 + x^2)^3}{6} + C}}$

57. RESOLUCIÓN

$$I = \int x\sqrt{2x^2 + 3}\cdot dx = \int t \cdot \frac{t\,dt}{2} = \frac{1}{2}\int t^2\, dt =$$

$$= \frac{1}{2}\cdot\frac{t^3}{3} + C = \frac{(2x^2 + 3)^{3/2}}{6} + C$$

CÁLCULOS AUXILIARES

$\sqrt{2x^2 + 3} = t \quad ; \quad 2x^2 + 3 = t^2$

$4x\,dx = 2t\,dt$

$$x\,dx = \frac{t\,dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{(2x^2 + 3)^{3/2}}{6} + C}}$

58. RESOLUCIÓN

$$I = \int \frac{4x^2\, dx}{\sqrt{x^3 + 8}} = 4\int \frac{x^2\, dx}{\sqrt{x^3 + 8}} = 4\int \frac{\frac{2t\,dt}{3}}{t} = \frac{8}{3}\int dt =$$

$$= \frac{8}{3}t + C = \frac{8\sqrt{x^3 + 8}}{3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x^3 + 8} = t \quad ; \quad x^3 + 8 = t^2$

$3x^2\,dx = 2t\,dt$

$$x^2\,dx = \frac{2t\,dt}{3}$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{8\sqrt{x^3 + 8}}{3} + C}}$

59. RESOLUCIÓN

$$I = \int \frac{(\sqrt{a} + \sqrt{x})^2}{\sqrt{x}}\, dx = \int t^2 \cdot 2\, dt = 2\int t^2\, dt = 2\cdot\frac{t^3}{3} + C =$$

$$= \frac{2\,(\sqrt{a} + \sqrt{x})^3}{3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{a} + \sqrt{x} = t$

$\dfrac{dx}{2\sqrt{x}} = dt$

$\dfrac{dx}{\sqrt{x}} = 2\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2\,(\sqrt{a} + \sqrt{x})^3}{3} + C}}$

60. RESOLUCIÓN

$$I = \int \frac{x^3\,dx}{\sqrt{a^4 - x^4}} = \int \frac{-\dfrac{t\,dt}{2}}{t} = -\frac{1}{2}\int dt = -\frac{1}{2}t + C =$$

$$= -\frac{\sqrt{a^4 - x^4}}{2} + C$$

CÁLCULOS AUXILIARES

$\sqrt{a^4 - x^4} = t \quad ; \quad a^4 - x^4 = t^2$

$-4x^3\,dx = 2t\,dt$

$x^3\,dx = -\dfrac{t\,dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{\sqrt{a^4 - x^4}}{2} + C}}$

61. RESOLUCIÓN

$$I = \int \frac{x\,dx}{(a + bx^2)^3} = \int \frac{\dfrac{dt}{2b}}{t^3} = \frac{1}{2b}\int t^{-3}\,dt = \frac{1}{2b}\cdot\frac{t^{-2}}{-2} + C$$

$$= -\frac{1}{4b\,t^2} + C = -\frac{1}{4b\,(a + bx^2)^2} + C$$

CÁLCULOS AUXILIARES

$a + bx^2 = t$

$2bx\,dx = dt$

$x\,dx = \dfrac{dt}{2b}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{4b\,(a + bx^2)^2} + C}}$

62. RESOLUCIÓN

$$I = \int x^{n-1}\sqrt{a + bx^n}\,dx = \int \frac{t \cdot 2t\,dt}{bn} = \frac{2}{bn}\int t^2\,dt =$$

$$= \frac{2}{bn}\cdot\frac{t^3}{3} + C = \frac{2\,(a + bx^n)^{3/2}}{3bn} + C$$

CÁLCULOS AUXILIARES

$\sqrt{a + bx^n} = t$

$a + bx^n = t^2$

$bn\,x^{n-1}\,dx = 2t\,dt$

$x^{n-1}\,dx = \dfrac{2t\,dt}{bn}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{2\,(a + bx^n)^{3/2}}{3bn} + C}}$

63. RESOLUCIÓN

$$I = \int \frac{(2x + 3)\,dx}{\sqrt{x^2 + 3x - 1}} = \int \frac{2t\,dt}{t} = 2\int dt = 2t + C =$$

$$= 2\sqrt{x^2 + 3x - 1} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x^2 + 3x - 1} = t$

$x^2 + 3x - 1 = t^2$

$(2x + 3)\,dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{x^2 + 3x - 1} + C}}$

64. RESOLUCIÓN

$$I = \int \frac{(x^2 + 1)\,dx}{\sqrt{x^3 + 3x}} = \int \frac{\frac{2t\,dt}{3}}{t} = \frac{2}{3}\int dt = \frac{2}{3}t + C =$$

$$= \frac{2\sqrt{x^3 + 3x}}{3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x^3 + 3x} = t$

$x^3 + 3x = t^2$

$(3x^2 + 3)\,dx = 2t\,dt$

$3(x^2 + 1)\,dx = 2t\,dt$

$(x^2 + 1)\,dx = \dfrac{2t\,dt}{3}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2\sqrt{x^3 + 3x}}{3} + C}}$

65. RESOLUCIÓN

$$I = \int \frac{(x + 5)\,dx}{\sqrt{x^2 + 10x - 4}} = \int \frac{t\,dt}{t} = \int dt = t + C =$$

$$= \sqrt{x^2 + 10x - 4} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x^2 + 10x - 4} = t$

$x^2 + 10x - 4 = t^2$

$(2x + 10)\,dx = 2t\,dt$

$2(x + 5)\,dx = 2t\,dt$

$(x + 5)\,dx = t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \sqrt{x^2 + 10 - 4} + C}}$

66. RESOLUCIÓN

$$I = \int \frac{2 + Lx}{x}\,dx = \int (2 + Lx)\,\frac{dx}{x} = \int t\,dt = \frac{t^2}{2} + C =$$

$$= \frac{(2 + Lx)^2}{2} + C$$

CÁLCULOS AUXILIARES

$2 + Lx = t$

$\dfrac{dx}{x} = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{(2 + Lx)^2}{2} + C}}$

67. RESOLUCIÓN

$$I = \int sen\,3x \cdot cos\,3x\,dx = \int t \cdot \frac{dt}{3} = \frac{1}{3}\int t\,dt =$$

$$= \frac{1}{3} \cdot \frac{t^2}{2} + C = \frac{sen^2\,3x}{6} + C$$

CÁLCULOS AUXILIARES

$sen\,3x = t$

$3\,cos\,3x\,dx = dt$

$cos\,3x\,dx = \dfrac{dt}{3}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{sen^2\,3x}{6} + C}}$

68. RESOLUCIÓN

$$I = \int (sen\,2x \cdot cos^2\,2x)\,dx = \int t^2 \cdot \frac{-dt}{2} = -\frac{1}{2}\int t^2\,dt =$$

$$= -\frac{1}{2} \cdot \frac{t^3}{3} + C = -\frac{cos^3\,2x}{6} + C$$

CÁLCULOS AUXILIARES

$\cos 2x = t$

$-2 \operatorname{sen} 2x\, dx = dt$

$\operatorname{sen} 2x\, dx = -\dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{\cos^3 2x}{6} + C}}$

69. RESOLUCIÓN

$$I = \int \operatorname{tg} \frac{x}{2} \cdot \sec^2 \frac{x}{2}\, dx = \int t \cdot 2\, dt = 2\int t\, dt = 2 \cdot \frac{t^2}{2} + C =$$

$$= \operatorname{tg}^2 \frac{x}{2} + C$$

CÁLCULOS AUXILIARES

$\operatorname{tg} \dfrac{x}{2} = t$

$\dfrac{1}{2} \sec^2 \dfrac{x}{2}\, dx = dt$

$\sec^2 \dfrac{x}{2}\, dx = 2\, dt$

SOLUCIÓN: $\boxed{\mathbf{I = tg^2 \dfrac{x}{2} + C}}$

70. RESOLUCIÓN

$$I = \int \frac{\cos 4x\, dx}{\sqrt{3 + \operatorname{sen} 4x}} = \int \frac{\frac{t\, dt}{2}}{t} = \frac{1}{2}\int dt = \frac{1}{2} \cdot t + C =$$

$$= \frac{\sqrt{3 + \operatorname{sen} 4x}}{2} + C$$

CÁLCULOS AUXILIARES

$\sqrt{3 + \operatorname{sen} 4x} = t$

$3 + \operatorname{sen} 4x = t^2$

$4 \cos 4x\, dx = 2t\, dt$

$\cos 4x\, dx = \dfrac{t\, dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{\sqrt{3 + sen\ 4x}}{2} + C}}$

71. RESOLUCIÓN

$$I = \int \left(\frac{\sec 2x}{1 + \operatorname{tg} 2x}\right)^2 dx = \int \frac{\sec^2 2x\, dx}{(1 + \operatorname{tg} 2x)^2} = \int \frac{\frac{dt}{2}}{t^2} =$$

$$= \frac{1}{2}\int t^{-2}\, dt = \frac{1}{2} \cdot \frac{t^{-1}}{-1} + C = -\frac{1}{2t} + C =$$

$$= -\frac{1}{2\,(1 + \operatorname{tg} 2x)} + C$$

CÁLCULOS AUXILIARES

$1 + \operatorname{tg} 2x = t$

$2 \sec^2 2x\, dx = dt$

$\sec^2 2x\, dx = \dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{-1}{2\,(1 + tg\ 2x)} + C}}$

72. RESOLUCIÓN

$$I = \int \frac{e^{2x}\, dx}{3 + 5e^{2x}} = \int \frac{\frac{dt}{10}}{t} = \frac{1}{10}\int \frac{dt}{t} = \frac{1}{10} \cdot L\,|t| + C =$$

$$= \frac{L\,|3 + 5e^{2x}|}{10} + C$$

CÁLCULOS AUXILIARES

$3 + 5e^{2x} = t$

$5 \cdot 2e^{2x}\,dx = dt$

$e^{2x}\,dx = \dfrac{dt}{10}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{L\,|3 + 5e^{2x}|}{10} + C}}$

73. RESOLUCIÓN

$$I = \int \frac{x\,dx}{\sqrt{1 - 2x^2}} = \int \frac{\dfrac{-t\,dt}{2}}{t} = -\frac{1}{2}\int dt = -\frac{1}{2}t + C =$$

$$= -\frac{\sqrt{1 - 2x^2}}{2} + C$$

CÁLCULOS AUXILIARES

$\sqrt{1 - 2x^2} = t$

$1 - 2x^2 = t^2$

$-4x\,dx = 2t\,dt$

$x\,dx = -\dfrac{t\,dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{\sqrt{1 - 2x^2}}{2} + C}}$

74. RESOLUCIÓN

$$I = \int \frac{sen\ ax\,dx}{cos\ ax + b} = \int \frac{-\dfrac{dt}{a}}{t} = -\frac{1}{a}\int \frac{dt}{t} =$$

$$= -\frac{1}{a}L\,|t| + C = -\frac{L\,|cos\ ax + b|}{a} + C$$

CÁLCULOS AUXILIARES

$cos\ ax + b = t$

$-a\ sen\ ax\,dx = dt$

$sen\ ax\,dx = -\dfrac{dt}{a}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{L\,|cos\ ax + b|}{a} + C}}$

75. RESOLUCIÓN

$$I = \int \frac{cosec^2 x\,dx}{\sqrt{2\ ctg\ x + 3}} = \int \frac{-t\,dt}{t} = -\int dt = -t + C =$$

$$= -\sqrt{2\ ctg\ x + 3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{2\ ctg\ x + 3} = t$

$2\ ctg\ x + 3 = t^2$

$2\,(-cosec^2 x)\,dx = 2t\,dt$

$cosec^2 x\,dx = -t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = -\sqrt{2\ ctg\ x + 3} + C}}$

76. RESOLUCIÓN

$$I = \int \frac{e^x + 2}{\sqrt{e^x + 2x}}\,dx = \int \frac{2t\,dt}{t} = 2\int dt = 2t + C =$$

$$= 2\sqrt{e^x + 2x} + C$$

CÁLCULOS AUXILIARES

$\sqrt{e^x + 2x} = t$

$e^x + 2x = t^2$

$(e^x + 2)\,dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{e^x + 2x} + C}}$

77. RESOLUCIÓN

$$I = \int \frac{e^x + \operatorname{sen} x}{\sqrt{e^x - \cos x}}\,dx = \int \frac{2t\,dt}{t} = 2\int dt = 2t + C =$$

$$= 2\sqrt{e^x - \cos x} + C$$

CÁLCULOS AUXILIARES

$$\sqrt{e^x - \cos x} = t$$

$$e^x - \cos x = t^2$$

$$(e^x + \operatorname{sen} x)\,dx = 2t\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{e^x - \cos x} + C}}$

78. RESOLUCIÓN

$$I = \int \frac{\sec 2x \operatorname{tg} 2x}{\sec 2x - 3}\,dx = \int \frac{\frac{dt}{2}}{t} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2} L|t| + C =$$

$$= \frac{L|\sec 2x - 3|}{2} + C$$

CÁLCULOS AUXILIARES

$$\sec 2x - 3 = t$$

$$2 \sec 2x \cdot \operatorname{tg} 2x\,dx = dt$$

$$\sec 2x \cdot \operatorname{tg} 2x\,dx = \frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{L|\sec 2x - 3|}{2} + C}}$

79. RESOLUCIÓN

$$I = \int (e^{x/a} - e^{-x/a})\,dx = \int e^{x/a}\,dx - \int e^{-x/a}\,dx =$$

$$= \int e^t \cdot a\,dt - \int e^z\,(-a)\,dz = a \cdot e^t + a \cdot e^z + C =$$

$$= a \cdot e^{x/a} + a \cdot e^{-x/a} + C = a\,(e^{x/a} + e^{-x/a}) + C$$

CÁLCULOS AUXILIARES

$$\frac{x}{a} = t \quad ; \quad x = at$$

$$dx = a\,dt$$

$$-\frac{x}{a} = z \quad ; \quad x = -az$$

$$dx = -a\,dz$$

SOLUCIÓN: $\boxed{\mathbf{I = a\,(e^{x/a} + e^{-x/a}) + C}}$

80. RESOLUCIÓN

$$I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^t}{t} \cdot 2t\,dt = 2\int e^t\,dt = 2e^t + C = 2e^{\sqrt{x}} + C$$

CÁLCULOS AUXILIARES

$$\sqrt{x} = t \quad ; \quad x = t^2$$

$$dx = 2t\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = 2e^{\sqrt{x}} + C}}$

81. RESOLUCIÓN

$$I = \int e^{\operatorname{tg} x} \cdot \sec^2 x\,dx = \int e^t\,dt = e^t + C = e^{\operatorname{tg} x} + C$$

CÁLCULOS AUXILIARES

$$\operatorname{tg} x = t$$

$$\sec^2 x\,dx = dt$$

SOLUCIÓN: $\boxed{\mathbf{I = e^{\operatorname{tg} x} + C}}$

82. RESOLUCIÓN

$$I = \int a^{2x}\,dx = \int a^t \frac{dt}{2} = \frac{1}{2}\int a^t\,dt = \frac{1}{2} \cdot \frac{a^t}{L\,a} + C =$$

$$= \frac{a^{2x}}{2\,L\,a} + C$$

CÁLCULOS AUXILIARES

$2x = t$

$2\,dx = dt$

$dx = \dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{a^{2x}}{2\,La} + C}}$

83. RESOLUCIÓN

$$I = \int (e^{5x} + a^{5x})\,dx = \int e^{5x}\,dx + \int a^{5x}\,dx =$$

$$= \int e^t \cdot \frac{dt}{5} + \int a^t \cdot \frac{dt}{5} = \frac{1}{5} e^t + \frac{1}{5} \cdot \frac{a^t}{L\,a} + C =$$

$$= \frac{e^{5x}}{5} + \frac{a^{5x}}{5\,L\,a} + C$$

CÁLCULOS AUXILIARES

$5x = t$

$5\,dx = dt$

$dx = \dfrac{dt}{5}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{e^{5x}}{5} + \dfrac{a^{5x}}{5\,La} + C}}$

84. RESOLUCIÓN

$$I = \int \frac{3\,dx}{e^{3x}} = \int \frac{dt}{e^t} = \int e^{-t}\,dt = -e^{-t} + C = -e^{-3x} + C$$

CÁLCULOS AUXILIARES

$3x = t$

$3\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = -e^{-3x} + C}}$

85. RESOLUCIÓN

$$I = \int 6x\,e^{-x^2}\,dx = 6 \int e^t \frac{-dt}{2} = -3 \int e^t\,dt = -3\,e^t + C =$$

$$= -3\,e^{-x^2} + C$$

CÁLCULOS AUXILIARES

$-x^2 = t$

$-2x\,dx = dt$

$x\,dx = -\dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -3e^{-x^2} + C}}$

86. RESOLUCIÓN

$$I = \int \frac{e^{\sqrt{x}} - 3}{\sqrt{x}}\,dx = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx - 3 \int \frac{dx}{\sqrt{x}} =$$

$$= \int \frac{e^t}{t} \cdot 2t\,dt - 3 \cdot 2\sqrt{x} = 2 \int e^t\,dt - 6\sqrt{x} = 2\,e^t - 6\sqrt{x} + C =$$

$$= 2\,e^{\sqrt{x}} - 6\sqrt{x} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x} = t$

$x = t^2 \quad ; \quad dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2\,e^{\sqrt{x}} - 6\sqrt{x} + C}}$

87. RESOLUCIÓN

$$I = \int sen\,\frac{2x}{3}\,dx = \int sen\,t \cdot \frac{3\,dt}{2} = \frac{3}{2} \int sen\,t\,dt =$$

$$= -\frac{3}{2} cos\,t + C = -\frac{3}{2} cos\,\frac{2x}{3} + C$$

CÁLCULOS AUXILIARES

$$\frac{2x}{3} = t$$

$$\frac{2}{3}\,dx = dt$$

$$dx = \frac{3\,dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{3}{2}\cos\frac{2x}{3} + C}}$

88. RESOLUCIÓN

$$I = \int \cos 5x\,dx = \int \cos t \cdot \frac{dt}{5} = \frac{1}{5}\int \cos t\,dt$$

$$= \frac{1}{5}\,\text{sen}\ t + C = \frac{\text{sen}\ 5x}{5} + C$$

CÁLCULOS AUXILIARES

$$5x = t$$

$$5\,dx = dt$$

$$dx = \frac{dt}{5}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{sen\ 5x}{5} + C}}$

89. RESOLUCIÓN

$$I = \int \text{tg}\,\frac{x}{5}\,dx = \int \text{tg}\ t \cdot 5\,dt = 5\int \text{tg}\ t\,dt = -5\,L\,|\cos t| + C =$$

$$= -5\,L\left|\cos\frac{x}{5}\right| + C$$

CÁLCULOS AUXILIARES

$$\frac{x}{5} = t\ ;\ x = 5t\ ;\ dx = 5\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = -5\,L\left|\cos\frac{x}{5}\right| + C}}$

90. RESOLUCIÓN

$$I = \int \text{ctg}\ 10x\,dx = \int \text{ctg}\ t \cdot \frac{dt}{10} = \frac{1}{10}\int \text{ctg}\ t\,dt =$$

$$= \frac{1}{10} \cdot L\,|\text{sen}\ t| + C = \frac{1}{10}\,L\,|\text{sen}\ 10x| + C$$

CÁLCULOS AUXILIARES

$$10x = t$$

$$10\,dx = dt$$

$$dx = \frac{dt}{10}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{10}\,L\,|sen\ 10x| + C}}$

91. RESOLUCIÓN

$$I = \int \text{cosec}\ x\,dx = \int \text{cosec}\ x \cdot \frac{\text{cosec}\ x + \text{ctg}\ x}{\text{cosec}\ x + \text{ctg}\ x}\,dx = -\int \frac{dt}{t} =$$

$$= -L\,|t| + C = -L\,|\text{cosec}\ x + \text{ctg}\ x| + C$$

CÁLCULOS AUXILIARES

$$\text{cosec}\ x + \text{ctg}\ x = t$$

$$(-\text{cosec}\ x \cdot \text{ctg}\ x - \text{cosec}^2\ x)\,dx = dt$$

$$-\text{cosec}\ x\,(\text{cosec}\ x + \text{ctg}\ x)\,dx = dt$$

$$\text{cosec}\ x\,(\text{cosec}\ x + \text{ctg}\ x)\,dx = -dt$$

SOLUCIÓN: $\boxed{\mathbf{I = -L\,|cosec\ x + ctg\ x| + C}}$

92. RESOLUCIÓN

$$I = \int \frac{dx}{(1 + x^2)\ \text{arc tg}\ x} = \int \frac{dt}{t} = L\,t + C = L\,|\text{arc tg}\ x| + C$$

CÁLCULOS AUXILIARES

$arc\ tg\ x = t$

$$\frac{dx}{1+x^2} = dt$$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|arc\ tg\ x| + C}}$

93. RESOLUCIÓN

$$I = \int \sqrt{\frac{1+x}{1-x}}\,dx = \int \frac{\sqrt{1+x}\cdot\sqrt{1+x}}{\sqrt{1-x}\cdot\sqrt{1+x}}\,dx = \int \frac{1+x}{\sqrt{1-x^2}}\,dx =$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x\,dx}{\sqrt{1-x^2}} = arc\ sen\ x + \int \frac{-\frac{dt}{2}}{\sqrt{t}} =$$

$$= arc\ sen\ x - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = arc\ sen\ x - \frac{1}{2}\cdot 2\sqrt{t} + C =$$

$$= arc\ sen\ x - \sqrt{1-x^2} + C$$

CÁLCULOS AUXILIARES

$1 - x^2 = t$

$-2x\,dx = dt$

$$x\,dx = -\frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ sen\ x - \sqrt{1-x^2} + C}}$

94. RESOLUCIÓN

$$I = \int \frac{dx}{x^2+9} = 3\int \frac{dt}{9t^2+9} = \frac{1}{3}\int \frac{dt}{t^2+1} =$$

$$= \frac{1}{3}\,arc\ tg\ t + C = \frac{1}{3}\,arc\ tg\ \frac{x}{3} + C$$

CÁLCULOS AUXILIARES

$$x = 3t \quad ; \quad t = \frac{x}{3}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{3}\,arc\ tg\ \frac{x}{3} + C}}$

95. RESOLUCIÓN

$$I = \int \frac{dx}{(x-2)^2+9} = 3\int \frac{dt}{9t^2+9} = \frac{1}{3}\int \frac{dt}{t^2+1} =$$

$$= \frac{1}{3}\,arc\ tg\ t + C = \frac{1}{3}\,arc\ tg\ \frac{x-2}{3} + C$$

CÁLCULOS AUXILIARES

$$x - 2 = 3t \quad ; \quad t = \frac{x-2}{3}$$

$dx = 3\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{3}\,arc\ tg\ \frac{x-2}{3} + C}}$

96. RESOLUCIÓN

$$I = \int \frac{ax\,dx}{x^4+b^4} = a\int \frac{x\,dx}{(x^2)^2+b^4} = a\int \frac{\frac{b^2\,dt}{2}}{b^4t^2+b^4} =$$

$$= \frac{a\,b^2}{2\,b^4}\int \frac{dt}{t^2+1} = \frac{a}{2\,b^2}\,arc\ tg\ t + C =$$

$$= \frac{a}{2\,b^2}\,arc\ tg\ \frac{x^2}{b^2} + C$$

CÁLCULOS AUXILIARES

$$x^2 = b^2\,t \Rightarrow t = \frac{x^2}{b^2}$$

$2x\,dx = b^2\,dt$

$$x\,dx = \frac{b^2\,dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{a}{2b^2}\,arc\ tg\ \frac{x^2}{b^2} + C}}$

97. RESOLUCIÓN

$$I = \int \frac{dx}{x\,Lx} = \int \frac{1}{Lx} \cdot \frac{dx}{x} = \int \frac{1}{t} dt = Lt + C = L\,(Lx) + C$$

CÁLCULOS AUXILIARES

$Lx = t$

$$\frac{dx}{x} = dt$$

SOLUCIÓN: $\boxed{\mathbf{I = L\,(Lx) + C}}$

98. RESOLUCIÓN

$$I = \int \cos^2 5x\, dx = \frac{1}{2}\int (1 + \cos 10x)\, dx =$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos 10x\, dx = \frac{1}{2}x + \frac{1}{2} \cdot \frac{sen\ 10x}{10} + C =$$

$$= \frac{x}{2} + \frac{sen\ 10x}{20} + C$$

CÁLCULOS AUXILIARES

$$sen^2 5x + \cos^2 5x = 1$$
$$\cos^2 5x - sen^2 5x = \cos 10x$$
$$2\cos^2 5x = 1 + \cos 10x$$
$$\cos^2 5x = \frac{1 + \cos 10x}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{2} + \frac{sen\ 10x}{20} + C}}$

99. RESOLUCIÓN

$$I = \int \cos x \cdot sen\ 2x\, dx = \int \cos x \cdot 2\, sen\ x \cos x\, dx =$$

$$= 2\int \cos^2 x \cdot sen\ x\, dx = -2\int t^2\, dt = -2 \cdot \frac{t^3}{3} + C =$$

$$= -\frac{2\cos^3 x}{3} + C$$

CÁLCULOS AUXILIARES

$\cos x = t$

$-sen\ x\, dx = dt$

$sen\ x\, dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{2\cos^3 x}{3} + C}}$

100. RESOLUCIÓN

$$I = \int \frac{dx}{1 + \cos x} = \int \frac{(1 - \cos x)}{(1 - \cos x)\,(1 + \cos x)} dx =$$

$$= \int \frac{(1 - \cos x)\, dx}{1 - \cos^2 x} = \int \frac{(1 - \cos x)\, dx}{sen^2 x} =$$

$$= \int \frac{dx}{sen^2 x} - \int \frac{\cos x}{sen^2 x} dx = -ctg\ x - \int \frac{dt}{t^2} =$$

$$= -ctg\ x - \frac{t^{-1}}{-1} + C = -ctg\ x + \frac{1}{sen\ x} + C$$

CÁLCULOS AUXILIARES

$sen\ x = t$

$\cos x\, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = -ctg\ x + \frac{1}{sen\ x} + C}}$

101. RESOLUCIÓN

$$I = \int sen^2 3x\, dx = \int \frac{1 - \cos 6x}{2} dx = \int \frac{dx}{2} - \int \frac{\cos 6x}{2} dx =$$

$$= \frac{x}{2} - \frac{1}{2} \cdot \frac{1}{6} sen\ 6x + C = \frac{x}{2} - \frac{1}{12} sen\ 6x + C$$

CÁLCULOS AUXILIARES

$$sen^2 3x + \cos^2 3x = 1$$
$$-\cos^2 3x + sen^2 3x = -\cos 6x$$
$$2\, sen^2 3x = 1 - \cos 6x$$
$$sen^2 3x = \frac{1 - \cos 6x}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{2} - \frac{1}{12}\ sen\ 6x + C}}$

102. RESOLUCIÓN

$$I = \int tg^2\, 2x\, dx = \int (sec^2\, 2x - 1)\, dx = \int \frac{dx}{cos^2\, 2x} - \int dx =$$

$$= \frac{1}{2}\, tg\, 2x - x + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{2}\ tg\ 2x - x + C}}$

103. RESOLUCIÓN

$$I = \int sec^2\, 10x\, dx = \int \frac{dx}{cos^2\, 10x} = \frac{1}{10}\, tg\, 10x + C$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{10}\ tg\ 10x + C}}$

104. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{25 - x^2}} = \int \frac{5\, dt}{\sqrt{25 - (5t)^2}} = \int \frac{5\, dt}{\sqrt{25 - 25t^2}} =$$

$$= \int \frac{dt}{\sqrt{1 - t^2}} = arc\ sen\ t + C = arc\ sen\ \frac{x}{5} + C$$

CÁLCULOS AUXILIARES

$$x = 5t \Rightarrow t = \frac{x}{5}$$

$$dx = 5\, dt$$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ sen\ \frac{x}{5} + C}}$

105. RESOLUCIÓN

$$I = \int \frac{5x\, dx}{\sqrt{1 - x^4}} = 5 \int \frac{x\, dx}{\sqrt{1 - (x^2)^2}} = 5 \int \frac{\frac{dt}{2}}{\sqrt{1 - t^2}} =$$

$$= \frac{5}{2} \int \frac{dt}{\sqrt{1 - t^2}} = \frac{5}{2}\, arc\ sen\ t + C = \frac{5}{2}\, arc\ sen\ x^2 + C$$

CÁLCULOS AUXILIARES

$$x^2 = t$$

$$2x\, dx = dt$$

$$x\, dx = \frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{5}{2}\ arc\ sen\ x^2 + C}}$

106. RESOLUCIÓN

$$I = \int sec^2 \left(\frac{8x^2 - 2x - 15}{4x + 5} \right) dx = \int sec^2\, (2x - 3)\, dx =$$

$$= \frac{1}{2} \int sec^2\, t\, dt = \frac{1}{2}\, tg\, t + C = \frac{1}{2}\, tg\, (2x - 3) + C$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} 8x^2 - 2x - 15 & 4x + 5 \\ \underline{-8x^2 - 10x} & \hline \\ -12x - 15 & 2x - 3 \\ \underline{12x + 15} & \\ 0 & \end{array}$$

$$2x - 3 = t$$

$$2x\, dx = dt$$

$$x\, dx = \frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{2}\ tg\ (2x - 3) + C}}$

$$I = \int \frac{1}{4} x\, a^{x^2}\, dx = \int \frac{1}{4} \cdot \frac{dt}{2\,La} = \frac{1}{8\,La} \int dt =$$

$$= \frac{1}{8\,La} \cdot t + C = \frac{a^{x^2}}{8\,La} + C$$

CÁLCULOS AUXILIARES

$a^{x^2} = t$

$a^{x^2} \cdot La \cdot 2x\, dx = dt$

$x\, a^{x^2} dx = \dfrac{dt}{2\,La}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{a^{x^2}}{8\,La} + C}}$

108. RESOLUCIÓN

$$I = \int \frac{3\,dx}{5^{2x-1}} = 3 \int 5^{1-2x}\, dx = -\frac{3}{2} \int 5^t\, dt =$$

$$= -\frac{3}{2} \cdot \frac{5^t}{L5} + C = -\frac{3 \cdot 5^{1-2x}}{2\,L5} + C$$

CÁLCULOS AUXILIARES

$1 - 2x = t$

$-2\,dx = dt$

$dx = -\dfrac{dt}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{3 \cdot 5^{1-2x}}{2\,L5} + C}}$

109. RESOLUCIÓN

$$I = \int \frac{x^6\,dx}{\cos^2(x^7+2)} = \frac{1}{7} \int \frac{dt}{\cos^2 t} = \frac{1}{7}\, tg\, t + C =$$

$$= \frac{1}{7}\, tg\,(x^7+2) + C$$

CÁLCULOS AUXILIARES

$x^7 + 2 = t$

$7x^6\,dx = dt$

$x^6\,dx = \dfrac{dt}{7}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{1}{7}\, tg\,(x^7+2) + C}}$

110. RESOLUCIÓN

$$I = \int \frac{\sqrt{x^2-1}}{x} dx = \int \frac{\sqrt{x^2-1} \cdot \sqrt{x^2-1}}{x \cdot \sqrt{x^2-1}} dx = \int \frac{x^2-1}{x\sqrt{x^2-1}} =$$

$$= \int \frac{x\,dx}{\sqrt{x^2-1}} - \int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{t\,dt}{t} - arc\,sec\,x =$$

$$= t - arc\,sec\,x + C = \sqrt{x^2-1} - arc\,sec\,x + C$$

CÁLCULOS AUXILIARES

$\sqrt{x^2-1} = t$

$x^2 - 1 = t^2$

$2x\,dx = 2t\,dt$

$x\,dx = t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \sqrt{x^2-1} - arc\,sec\,x + C}}$

111. RESOLUCIÓN

$$I = \int \frac{x \cdot a^{\sqrt{x^2-1}}}{\sqrt{x^2-1}} dx = \int \frac{dt}{La} = \frac{1}{La} \int dt = \frac{1}{La} \cdot t + C =$$

$$= \frac{a^{\sqrt{x^2-1}}}{La} + C$$

CÁLCULOS AUXILIARES

$a^{\sqrt{x^2-1}} = t$

$$a^{\sqrt{x^2-1}} \cdot La \cdot \frac{1}{2\sqrt{x^2-1}} \cdot 2x\,dx = dt$$

$$\frac{x \cdot a^{\sqrt{x^2-1}}}{\sqrt{x^2-1}}\,dx = \frac{dt}{La}$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{a^{\sqrt{x^2-1}}}{La} + C}}$

112. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{4-(x+3)^2}} = 2\int \frac{dt}{\sqrt{4-(2t)^2}} = 2\int \frac{dt}{\sqrt{4-4t^2}} =$$

$$= \int \frac{dt}{\sqrt{1-t^2}} = \text{arc sen } t + C = \text{arc sen } \frac{x+3}{2} + C$$

CÁLCULOS AUXILIARES

$$x + 3 = 2t \Rightarrow t = \frac{x+3}{2}$$

$dx = 2\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ sen\ \dfrac{x+3}{2} + C}}$

113. RESOLUCIÓN

$$I = \int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(x+1)^2+4} = \frac{1}{2}\text{arc tg } \frac{x+1}{2} + C$$

CÁLCULOS AUXILIARES

$x^2 + 2x + 5 = 0$

$x_1 = -1 + 2i$

$x_2 = -1 - 2i$

$x^2 + 2x + 5 = (x+1)^2 + 4$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{1}{2}\ arc\ tg\ \dfrac{x+1}{2} + C}}$

114. RESOLUCIÓN

$$I = \int \frac{dx}{x^2+64} = \int \frac{8\,dt}{(8t)^2+64} = \frac{8}{64}\int \frac{dt}{t^2+1} =$$

$$= \frac{1}{8}\text{arc tg } t + C = \frac{1}{8}\text{arc tg } \frac{x}{8} + C$$

CÁLCULOS AUXILIARES

$$x = 8t \Rightarrow t = \frac{x}{8}$$

$dx = 8\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{1}{8}\ arc\ tg\ \dfrac{x}{8} + C}}$

115. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{16-9x^2}} = \frac{4}{3}\int \frac{dt}{\sqrt{16-9\left(\frac{4}{3}t\right)^2}} =$$

$$= \frac{4}{3}\int \frac{dt}{\sqrt{16-16t^2}} = \frac{1}{3}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{3}\text{arc sen } t + C =$$

$$= \frac{1}{3}\text{arc sen } \frac{3x}{4} + C$$

CÁLCULOS AUXILIARES

$$x = \frac{4}{3}t \Rightarrow t = \frac{3x}{4}$$

$$dx = \frac{4}{3}\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{1}{3}\ arc\ sen\ \dfrac{3x}{4} + C}}$

116. RESOLUCIÓN

$$I = \int \frac{e^x\,dx}{1 + e^{2x}} = \int \frac{e^x\,dx}{1 + (e^x)^2} = \int \frac{dt}{1 + t^2} =$$

$$= \text{arc tg } t + C = \text{arc tg } e^x + C$$

CÁLCULOS AUXILIARES

$e^x = t$

$e^x\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ tg\ e^x + C}}$

117. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{1 - a^2 x^2}} = \int \frac{\frac{1}{a}\,dt}{\sqrt{1 - a^2 \cdot \frac{t^2}{a^2}}} = \frac{1}{a}\int \frac{dt}{\sqrt{1 - t^2}} =$$

$$= \frac{1}{a}\,\text{arc sen } t + C = \frac{1}{a}\,\text{arc sen } ax + C$$

CÁLCULOS AUXILIARES

$$x = \frac{1}{a}\,t \Rightarrow t = ax$$

$$dx = \frac{1}{a}\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{a}\ arc\ sen\ ax + C}}$

118. RESOLUCIÓN

$$I = \int \frac{dx}{x\sqrt{1 - (Lx)^2}} = \int \frac{dt}{\sqrt{1 - t^2}} = \text{arc sen } t + C =$$

$$= \text{arc sen } (Lx) + C$$

CÁLCULOS AUXILIARES

$Lx = t$

$$\frac{dx}{x} = dt$$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ sen\ (Lx) + C}}$

119. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{2x - x^2}} = \int \frac{dx}{\sqrt{1 - (x - 1)^2}} = \int \frac{dt}{\sqrt{1 - t^2}} =$$

$$= \text{arc sen } t + C = \text{arc sen } (x - 1) + C$$

CÁLCULOS AUXILIARES

$x - 1 = t$

$dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ sen\ (x - 1) + C}}$

120. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{1 + 4x - x^2}} = \int \frac{dx}{\sqrt{5 - (x - 2)^2}} = \frac{1}{\sqrt{5}}\int \frac{\sqrt{5}\,dt}{\sqrt{1 - t^2}} =$$

$$= \text{arc sen } t + C = \text{arc sen } \frac{x - 2}{\sqrt{5}} + C$$

CÁLCULOS AUXILIARES

$$x - 2 = \sqrt{5}\,t \Rightarrow t = \frac{x - 2}{\sqrt{5}}$$

$$dx = \sqrt{5}\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ sen\ \frac{x - 2}{\sqrt{5}} + C}}$

121. RESOLUCIÓN

$$I = \int \frac{dx}{4x^2 + 25} = \frac{1}{25}\int \frac{dx}{\frac{4x^2}{25} + 1} = \frac{1}{25}\int \frac{dx}{\left(\frac{2}{5}x\right)^2 + 1} =$$

$$= \frac{1}{25}\int \frac{\frac{5}{2}\,dt}{t^2 + 1} = \frac{1}{10}\,arc\ tg\ t + C = \frac{1}{10}\,arc\ tg\ \frac{2x}{5} + C$$

CÁLCULOS AUXILIARES

$$\frac{2}{5}x = t\ ;\ x = \frac{5}{2}t$$

$$dx = \frac{5}{2}\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{10}\ arc\ tg\ \frac{2x}{5} + C}}$

122. RESOLUCIÓN

$$I = \int \frac{dx}{3 + 7x^2} = \frac{1}{3}\int \frac{dx}{1 + \frac{7}{3}x^2} = \frac{1}{3}\int \frac{dx}{1 + \left(\frac{\sqrt{7}}{\sqrt{3}}x\right)^2} =$$

$$= \frac{1}{3}\int \frac{\frac{\sqrt{3}}{\sqrt{7}}\,dt}{1 + t^2} = \frac{\sqrt{3}}{3\sqrt{7}}\,arc\ tg\ t + C =$$

$$= \frac{7\sqrt{3}}{21}\,arc\ tg\ \frac{\sqrt{7}}{\sqrt{3}}x + C$$

CÁLCULOS AUXILIARES

$$\frac{\sqrt{7}}{\sqrt{3}}x = t\ ;\ x = \frac{\sqrt{3}}{\sqrt{7}}t$$

$$dx = \frac{\sqrt{3}}{\sqrt{7}}\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{7\sqrt{3}}{21}\ arc\ tg\ \frac{\sqrt{7}}{\sqrt{3}}x + C}}$

123. RESOLUCIÓN

$$I = \int \frac{3\,dx}{x^2 - 8x + 25} = 3\int \frac{dx}{9 + (x - 4)^2} = 3\int \frac{3\,dt}{9 + 9t^2} =$$

$$= \int \frac{dt}{1 + t^2} = arc\ tg\ t + C = arc\ tg\ \frac{x - 4}{3} + C$$

CÁLCULOS AUXILIARES

$$x - 4 = 3t\ ;\ t = \frac{x - 4}{3}$$

$$dx = 3\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ tg\ \frac{x - 4}{3} + C}}$

124. RESOLUCIÓN

$$I = \int \frac{(2x + 5)}{x^2 + 2x + 5}dx = \int \frac{(2x + 2)\,dx}{x^2 + 2x + 5} + 3\int \frac{dx}{x^2 + 2x + 5} =$$

$$= L\,|x^2 + 2x + 5| + 3\int \frac{dx}{(x + 1)^2 + 4} = L\,|x^2 + 2x + 5| +$$

$$+ 3\int \frac{2\,dt}{4t^2 + 4} = L\,|x^2 + 2x + 5| + \frac{3}{2}\int \frac{dt}{t^2 + 1} =$$

$$= L\,|x^2 + 2x + 5| + \frac{3}{2}\,arc\ tg\ t + C =$$

$$= L\,|x^2 + 2x + 5| + \frac{3}{2}\,arc\ tg\ \frac{x + 1}{2} + C$$

CÁLCULOS AUXILIARES

$$x + 1 = 2t\ ;\ t = \frac{x + 1}{2}$$

$$dx = 2\,dt$$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|x^2 + 2x + 5| + \frac{3}{2}\,arc\,tg\,\frac{x+1}{2} + C}}$

25. RESOLUCIÓN

$$= \int \frac{(8x - 3)\,dx}{\sqrt{12x - 4x^2 - 5}} = \int \frac{(8x - 3 + 12 - 12)\,dx}{\sqrt{12x - 4x^2 - 5}} =$$

$$\int \frac{(8x - 12)\,dx}{\sqrt{12x - 4x^2 - 5}} + \int \frac{9\,dx}{\sqrt{12x - 4x^2 - 5}} =$$

$$- \int \frac{dt}{\sqrt{t}} + 9 \int \frac{dx}{\sqrt{4 - (2x - 3)^2}} =$$

$$= -2\sqrt{t} + 9 \int \frac{dz}{\sqrt{4 - 4z^2}} = -2\sqrt{12x - 4x^2 - 5} + \int \frac{9}{2}\,\frac{dz}{\sqrt{1 - z^2}}$$

$$= -2\sqrt{12x - 4x^2 - 5} + \frac{9}{2}\,arc\,sen\,z + C =$$

$$= -2\sqrt{12x - 4x^2 - 5} + \frac{9}{2}\,arc\,sen\,\frac{2x - 3}{2} + C$$

CÁLCULOS AUXILIARES

$12x - 4x^2 - 5 = t$

$(12 - 8x)\,dx = dt$

$-(8x - 12)\,dx = dt$

$(8x - 12)\,dx = -dt$

$12x - 4x^2 - 5 = 4 - (2x - 3)^2$

$2x - 3 = 2z \;;\; z = \frac{2x - 3}{2}$

$2\,dx = 2\,dz$

$dx = dz$

SOLUCIÓN: $\boxed{\mathbf{I = -2\sqrt{12x - 4x^2 - 5} + \frac{9}{2}\,arc\,sen\,\frac{2x - 3}{2} + C}}$

126. RESOLUCIÓN

$$I = \int sec\,x\,dx = \int sec\,x \cdot \frac{sec\,x + tg\,x}{sec\,x + tg\,x}\,dx =$$

$$= \int \frac{sec^2 x + sec\,x \cdot tg\,x}{sec\,x + tg\,x}\,dx = \int \frac{dt}{t} =$$

$$= L\,|t| + C = L\,|sec\,x + tg\,x| + C$$

CÁLCULOS AUXILIARES

$sec\,x + tg\,x = t$

$(sec\,x \cdot tg\,x + sec^2 x)\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|sec\,x + tg\,x| + C}}$

127. RESOLUCIÓN

$$I = \int \frac{cosec\,2x \cdot ctg\,2x}{5 - 4\,cosec\,2x}\,dx = \int \frac{\frac{dt}{8}}{t} = \frac{1}{8} \int \frac{dt}{t} =$$

$$= \frac{1}{8}\,L\,|t| + C = \frac{1}{8}\,L\,|5 - 4\,cosec\,2x| + C$$

CÁLCULOS AUXILIARES

$5 - 4\,cosec\,2x = t$

$-4\,(-cosec\,2x \cdot ctg\,2x \cdot 2)\,dx = dt$

$(cosec\,2x \cdot ctg\,2x)\,dx = \frac{dt}{8}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{8}\,L\,|5 - 4\,cosec\,2x| + C}}$

128. RESOLUCIÓN

$$I = \int \frac{\operatorname{cosec}^2 \frac{x}{2}\, dx}{\sqrt{3 - \operatorname{ctg} \frac{x}{2}}} = 2\int \frac{dt}{\sqrt{t}} = 2 \cdot 2\sqrt{t} + C =$$

$$= 4\sqrt{3 - \operatorname{ctg} \frac{x}{2}} + C$$

CÁLCULOS AUXILIARES

$$3 - \operatorname{ctg} \frac{x}{2} = t$$

$$\operatorname{cosec}^2 \frac{x}{2} \cdot \frac{1}{2}\, dx = dt$$

$$\operatorname{cosec}^2 \frac{x}{2}\, dx = 2\, dt$$

SOLUCIÓN: $\boxed{\mathbf{I = 4\sqrt{3 - ctg \frac{x}{2}} + C}}$

129. RESOLUCIÓN

$$I = \int \frac{dx}{\operatorname{sen} x \cdot \cos x} = \int \frac{\cos x\, dx}{\operatorname{sen} x \cdot \cos^2 x} = \int \operatorname{ctg} x \frac{dx}{\cos^2 x} =$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\operatorname{tg} x} = \int \frac{dt}{t} = L\,|t| + C = L\,|\operatorname{tg} x| + C$$

CÁLCULOS AUXILIARES

$$\operatorname{tg} x = t$$

$$\frac{dx}{\cos^2 x} = dt$$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|tg\,x| + C}}$

130. RESOLUCIÓN

$$I = \int x\, e^{-x}\, dx = -x\, e^{-x} + \int e^{-x}\, dx = -x\, e^{-x} - e^{-x} + C =$$

$$= -e^{-x}(x + 1) + C$$

CÁLCULOS AUXILIARES

$$u = x \;;\; du = dx$$

$$dv = e^{-x}\, dx \;;\; v = \int e^{-x}\, dx = -e^{-x}$$

SOLUCIÓN: $\boxed{\mathbf{I = -e^{-x}(x + 1) + C}}$

131. RESOLUCIÓN

$$I = \int \frac{Lx}{x^2}\, dx = -\frac{1}{x} \cdot Lx + \int \frac{dx}{x^2} =$$

$$= -\frac{Lx}{x} - \frac{1}{x} + C = -\frac{1}{x}(1 + Lx) + C$$

CÁLCULOS AUXILIARES

$$u = Lx \;;\; du = \frac{dx}{x}$$

$$dv = \frac{dx}{x^2} \;;\; v = \int \frac{dx}{x^2} = -\frac{1}{x}$$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{1}{x}(1 + L\,x) + C}}$

132. RESOLUCIÓN

$$I = \int x \cos x\, dx = x \operatorname{sen} x - \int \operatorname{sen} x\, dx = x \operatorname{sen} x + \cos x + C$$

CÁLCULOS AUXILIARES

$$u = x \;;\; du = dx$$

$$dv = \cos x\, dx \;;\; v = \int \cos x\, dx = \operatorname{sen} x$$

SOLUCIÓN: $\boxed{\mathbf{I = x\,sen\,x + cos\,x + C}}$

133. RESOLUCIÓN

$$I = \int x\, e^{ax}\, dx = x\,\frac{e^{ax}}{a} - \frac{1}{a}\int e^{ax}\, dx =$$

$$= \frac{x\, e^{ax}}{a} - \frac{1}{a}\cdot\frac{e^{ax}}{a} + C = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right) + C$$

CÁLCULOS AUXILIARES

$u = x \;;\; du = dx$

$dv = e^{ax}\, dx \;;\; v = \int e^{ax}\, dx = \dfrac{e^{ax}}{a}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{e^{ax}}{a}\left(x - \dfrac{1}{a}\right) + C}}$

134. RESOLUCIÓN

$$I = \int x^2\, e^{ax}\, dx = \frac{x^2\, e^{ax}}{a} - \frac{2}{a}\int x\, e^{ax}\, dx =$$

$$= \frac{x^2\, e^{ax}}{a} - \frac{2}{a}\left(\frac{x\, e^{ax}}{a} - \frac{1}{a}\int e^{ax}\, dx\right) =$$

$$= \frac{x^2\, e^{ax}}{a} - \frac{2}{a}\left(\frac{x\, e^{ax}}{a} - \frac{e^{ax}}{a^2}\right) + C =$$

$$= \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right) + C$$

CÁLCULOS AUXILIARES

$u = x^2 \;;\; du = 2x\, dx$

$dv = e^{ax}\, dx \;;\; v = \int e^{ax}\, dx = \dfrac{e^{ax}}{a}$

$u = x \;;\; du = dx$

$dv = e^{ax}\, dx \;;\; v = \int e^{ax}\, dx = \dfrac{e^{ax}}{a}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{e^{ax}}{a}\left(x^2 - \dfrac{2x}{a} + \dfrac{2}{a^2}\right) + C}}$

135. RESOLUCIÓN

$$I = \int L\, 4x\, dx = x\, L\, 4x - \int x\,\frac{dx}{x} =$$

$$= x\, L\, 4x - \int dx = x\, L\, 4x - x + C = x\,(L\, 4x - 1) + C$$

CÁLCULOS AUXILIARES

$u = L\, 4x \;;\; du = \dfrac{4}{4x}\, dx = \dfrac{dx}{x}$

$dv = dx \;;\; v = x$

SOLUCIÓN: $\boxed{\mathbf{I = x\,(L\,4x - 1) + C}}$

136. RESOLUCIÓN

$$I = \int x\, cos\, 4x\, dx = x\cdot\frac{sen\, 4x}{4} - \frac{1}{4}\int sen\, 4x\, dx =$$

$$= \frac{x}{4}\cdot sen\, 4x + \frac{1}{4}\cdot\frac{cos\, 4x}{4} + C =$$

$$= \frac{x}{4}\, sen\, 4x + \frac{1}{4^2}\, cos\, 4x + C$$

CÁLCULOS AUXILIARES

$u = x \;;\; du = dx$

$dv = cos\, 4x\, dx \;;\; v = \int cos\, 4x\, dx = \dfrac{sen\, 4x}{4}$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{x}{4}\, sen\, 4x + \dfrac{1}{16}\, cos\, 4x + C}}$

137. RESOLUCIÓN

$$I = \int x\, sec^2\, 3x\, dx = x\cdot\frac{tg\, 3x}{3} - \frac{1}{3}\int tg\, 3x\, dx =$$

$$= x\cdot\frac{tg\, 3x}{3} + \frac{1}{3}\cdot\frac{1}{3}\, L\,|cos\, 3x| + C =$$

$$= \frac{x\, tg\, 3x}{3} + \frac{1}{9}\, L\,|cos\, 3x| + C$$

CÁLCULOS AUXILIARES

$u = x \;;\; du = dx$

$dv = sec^2\, 3x\, dx \;;\; v = \int sec^2\, 3x\, dx = \dfrac{tg\, 3x}{3}$

$$\int tg\, 3x\, dx = \int \frac{sen\, 3x}{cos\, 3x}\, dx = -\frac{1}{3} L\, |cos\, 3x|$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{x\ tg\ 3x}{3} + \dfrac{1}{9} L\, |cos\ 3x| + C}}$

138. RESOLUCIÓN

$$I = \int arc\ cos\ x\, dx = x\ arc\ cos\ x - \int \frac{-x\, dx}{\sqrt{1 - x^2}} =$$

$$= x\ arc\ cos\ x + \int \frac{x\, dx}{\sqrt{1 - x^2}} = x\ arc\ cos\ x + (-\sqrt{1 - x^2}) + C =$$

$$= x\ arc\ cos\ x - \sqrt{1 - x^2} + C$$

CÁLCULOS AUXILIARES

$$u = arc\ cos\ x \;;\; du = -\frac{dx}{\sqrt{1 - x^2}}$$

$dv = dx \;;\; v = x$

$$\int \frac{x\, dx}{\sqrt{1 - x^2}} = -\int \frac{-2x\, dx}{2\sqrt{1 - x^2}} = -\sqrt{1 - x^2}$$

SOLUCIÓN: $\boxed{\mathbf{I = x\ arc\ cos\ x - \sqrt{1 - x^2} + C}}$

139. RESOLUCIÓN

$$I = \int arc\ tg\ x\, dx = x\ arc\ tg\ x - \int \frac{x\, dx}{1 + x^2} =$$

$$= x\ arc\ tg\ x - \frac{1}{2}\int \frac{2x\, dx}{1 + x^2} = x\ arc\ tg\ x - \frac{1}{2} L\, |1 + x^2| + C$$

CÁLCULOS AUXILIARES

$$u = arc\ tg\ x \;;\; du = \frac{dx}{1 + x^2}$$

$dv = dx \;;\; v = x$

SOLUCIÓN: $\boxed{\mathbf{I = x\ arc\ tg\ x - \dfrac{1}{2} L\, |1 + x^2| + C}}$

140. RESOLUCIÓN

$$I = \int arc\ ctg\ 3x\, dx = x\ arc\ ctg\ x - \int \frac{-3x\, dx}{1 + (3x)^2} =$$

$$= x\ arc\ ctg\ x + \frac{1}{2} L\, |1 + (3x)^2| + C$$

CÁLCULOS AUXILIARES

$$u = arc\ ctg\ 3x \;;\; du = -\frac{3\, dx}{1 + (3x)^2}$$

$dv = dx \;;\; v = x$

SOLUCIÓN: $\boxed{\mathbf{I = x\ arc\ ctg\ x + \dfrac{1}{2} L\, |1 + (3x)^2| + C}}$

141. RESOLUCIÓN

$$I = \int x^2\, Lx\, dx = \frac{x^3}{3} Lx - \frac{1}{3}\int x^3 \cdot \frac{dx}{x} = \frac{x^3}{3} Lx - \frac{1}{3}\int x^2\, dx =$$

$$= \frac{x^3}{3} Lx - \frac{1}{3} \cdot \frac{x^3}{3} + C = \frac{x^3}{3}\left(Lx - \frac{1}{3}\right) + C$$

CÁLCULOS AUXILIARES

$$u = Lx \;;\; du = \frac{dx}{x}$$

$$dv = x^2\, dx \;;\; v = \int x^2\, dx = \frac{x^3}{3}$$

SOLUCIÓN: $\boxed{\mathbf{I = \dfrac{x^3}{3}\left(Lx - \dfrac{1}{3}\right) + C}}$

142. RESOLUCIÓN

$$I = \int sen^2 x\, dx = \int sen\, x \cdot sen\, x\, dx = -sen\, x \cos x +$$

$$+ \int \cos^2 x\, dx = -sen\, x \cos x + \int (1 - sen^2 x)\, dx =$$

$$= -sen\, x \cos x + \int dx - \int sen^2 x\, dx = -sen\, x \cos x + x - I$$

$$2I = -sen\, x \cos x + x \Rightarrow I = \frac{1}{2}(x - sen\, x \cos x) + C$$

CÁLCULOS AUXILIARES

$u = sen\, x\ ;\ du = \cos x\, dx$

$dv = sen\, x\, dx\ ;\ v = \int sen\, x\, dx = -\cos x$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{2}(x - sen\, x \cos x) + C}}$

143. RESOLUCIÓN

$$I = \int x^2 \cos x\, dx = x^2\, sen\, x - 2\int x\, sen\, x\, dx =$$

$$= x^2\, sen\, x - 2\,[-x \cos x + \int \cos x\, dx] =$$

$$= x^2\, sen\, x + 2x \cos x - 2\int \cos x\, dx =$$

$$= x^2\, sen\, x + 2x \cos x - 2\, sen\, x + C$$

CÁLCULOS AUXILIARES

$u = x^2\ ;\ du = 2x\, dx$

$dv = \cos x\, dx\ ;\ v = \int \cos x\, dx = sen\, x$

$u = x\ ;\ du = dx$

$dv = sen\, x\, dx\ ;\ v = \int sen\, x\, dx = -\cos x$

SOLUCIÓN: $\boxed{\mathbf{I = x^2\, sen\, x + 2x \cos x - 2\, sen\, x + C}}$

144. RESOLUCIÓN

$$I = \int x\, arc\, tg\, x\, dx = \frac{x^2}{2} arc\, tg\, x - \frac{1}{2}\int \frac{x^2\, dx}{1 + x^2} =$$

$$= \frac{x^2}{2} arc\, tg\, x - \frac{1}{2}\int \frac{1 + x^2 - 1}{1 + x^2} dx =$$

$$= \frac{x^2}{2} arc\, tg\, x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1 + x^2} =$$

$$= \frac{x^2}{2} arc\, tg\, x - \frac{1}{2} x + \frac{1}{2} arc\, tg\, x + C =$$

$$= \left(\frac{x^2}{2} + \frac{1}{2}\right) arc\, tg\, x - \frac{x}{2} + C =$$

$$= \frac{x^2 + 1}{2} \cdot arc\, tg\, x - \frac{x}{2} + C$$

CÁLCULOS AUXILIARES

$u = arc\, tg\, x\ ;\ du = \frac{dx}{1 + x^2}$

$dv = x\, dx\ ;\ v = \int x\, dx = \frac{x^2}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^2 + 1}{2}\, arc\, tg\, x - \frac{x}{2} + C}}$

145. RESOLUCIÓN

$$I = \int x^2 e^{-x}\, dx = -x^2 e^{-x} + 2\int x\, e^{-x}\, dx =$$

$$= -x^2 e^{-x} + 2\,[-x\, e^{-x} + \int e^{-x}\, dx] =$$

$$= -x^2 e^{-x} - 2x\, e^{-x} - 2\, e^{-x} + C = -e^{-x}(x^2 + 2x + 2) + C$$

CÁLCULOS AUXILIARES

$u = x^2\ ;\ du = 2x\, dx$

$dv = e^{-x} dx\ ;\ v = \int e^{-x} dx = -e^{-x}$

$u = x\ ;\ du = dx$

$dv = e^{-x} dx\ ;\ v = -e^{-x}$

SOLUCIÓN: $\boxed{\mathbf{I = -e^{-x}(x^2 + 2x + 2) + C}}$

146. RESOLUCIÓN

$$I = \int (Lx)^2\, dx = x\,(Lx)^2 - 2\int Lx\, dx = x\,(Lx)^2 - 2\,[x\, Lx - \int dx] =$$

$$= x\,(Lx)^2 - 2x\, Lx + 2x + C$$

CÁLCULOS AUXILIARES

$u = (Lx)^2 \;;\; du = \frac{2}{x} Lx\, dx$ $\qquad$ $u = Lx \;;\; du = \frac{dx}{x}$

$dv = dx \;;\; v = x$ $\qquad$ $dv = dx \;;\; v = x$

SOLUCIÓN: $\boxed{\mathbf{I = x\,(Lx)^2 - 2x\,Lx + 2x + C}}$

147. RESOLUCIÓN

$$I = \int x \operatorname{sen} x \cos x\, dx = \frac{1}{2}\int x \cdot 2 \operatorname{sen} x \cos x\, dx =$$

$$= \frac{1}{2}\int x \cdot \operatorname{sen} 2x\, dx = \frac{1}{2}\left[-\frac{x \cdot \cos 2x}{2} + \frac{1}{2}\int \cos 2x\, dx\right] =$$

$$= -\frac{x \cdot \cos 2x}{4} + \frac{1}{4}\int \cos 2x\, dx = -\frac{x \cdot \cos 2x}{4} + \frac{\operatorname{sen} 2x}{8} + C$$

CÁLCULOS AUXILIARES

$u = x \;;\; du = dx$

$dv = \operatorname{sen} 2x\, dx \;;\; v = -\frac{\cos 2x}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{x \cos 2x}{4} + \frac{\operatorname{sen} 2x}{8} + C}}$

148. RESOLUCIÓN

$$I = \int \cos x \cdot L \operatorname{sen} x\, dx = \operatorname{sen} x \cdot L \operatorname{sen} x - \int \operatorname{sen} x \cdot \frac{\cos x}{\operatorname{sen} x} dx =$$

$$= \operatorname{sen} x\, L \operatorname{sen} x - \int \cos x\, dx = \operatorname{sen} x \cdot L \operatorname{sen} x - \operatorname{sen} x + C =$$

$$= \operatorname{sen} x\,(L \operatorname{sen} x - 1) + C$$

CÁLCULOS AUXILIARES

$u = L \operatorname{sen} x \;;\; du = \frac{\cos x\, dx}{\operatorname{sen} x}$

$dv = \cos x \;;\; v = \operatorname{sen} x$

SOLUCIÓN: $\boxed{\mathbf{I = sen\ x\,(L\ sen\ x - 1) + C}}$

149. RESOLUCIÓN

$$I = \int e^x \cdot \cos x\, dx = e^x \cdot \cos x + \int e^x \cdot \operatorname{sen} x\, dx =$$

$$= e^x \cos x + e^x \operatorname{sen} x - \int e^x \cos x\, dx = e^x \cos x + e^x \operatorname{sen} x - I$$

$$2I = e^x\,(\cos x + \operatorname{sen} x)$$

$$I = \frac{e^x}{2}\,(\cos x + \operatorname{sen} x) + C$$

CÁLCULOS AUXILIARES

$u = \cos x \;;\; du = -\operatorname{sen} x\, dx$

$dv = e^x\, dx \;;\; v = \int e^x\, dx = e^x$

$u = \operatorname{sen} x \;;\; du = \cos x\, dx$

$dv = e^x\, dx \;;\; v = e^x$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{e^x}{2}\,(cos\ x + sen\ x) + C}}$

150. RESOLUCIÓN

$$I = \int \frac{Lx}{(x+1)^2} dx = -\frac{1}{x+1} \cdot Lx - \int -\frac{dx}{x^2 + x} =$$

$$= -\frac{Lx}{x+1} + L\frac{x}{x+1} + C$$

CÁLCULOS AUXILIARES

$u = Lx \;;\; du = \frac{dx}{x}$

$dv = \frac{dx}{(x+1)^2} \;;$

$$v = \int \frac{dx}{(x+1)^2} = \int (x+1)^{-2}\, dx = \frac{(x+1)^{-1}}{-1} = -\frac{1}{x+1}$$

$$\int \frac{dx}{x^2 + x} = \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}} = \int \frac{dt}{t^2 - a^2} =$$

$$= \frac{1}{2a}\int\left(\frac{1}{t-a} - \frac{1}{t+a}\right)dt = \frac{1}{2a}L\frac{t-a}{t+a} = L\ \frac{x}{x+1}$$

$$x + \frac{1}{2} = t \Rightarrow dx = dt$$

$$\frac{1}{2} = a$$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{Lx}{x+1} + L\frac{x}{x+1} + C}}$

151. RESOLUCIÓN

$$I = \int\frac{dx}{(x^2+1)^2} = \int\frac{1+x^2-x^2}{(x^2+1)^2}dx = \int\frac{1+x^2}{(x^2+1)^2}dx -$$

$$-\int\frac{x^2}{(x^2+1)^2}dx = \int\frac{dx}{x^2+1} - \int\frac{x^2\,dx}{(x^2+1)^2} =$$

$$= arc\ tg\ x - \left[\frac{-x}{2(x^2+1)} + \frac{1}{2}\int\frac{dx}{1+x^2}\right] =$$

$$= arc\ tg\ x + \frac{x}{2(x^2+1)} - \frac{1}{2}arc\ tg\ x + C =$$

$$= \frac{1}{2}arc\ tg\ x + \frac{x}{2(x^2+1)} + C$$

CÁLCULOS AUXILIARES

$u = x\ ;\ du = dx$

$$dv = \frac{x\,dx}{(1+x^2)^2}\ ;$$

$$v = \int\frac{x\,dx}{(1+x^2)^2} = \frac{1}{2}\int\frac{dt}{t^2} = \frac{1}{2}\cdot\frac{-1}{t} = -\frac{1}{2(1+x^2)}$$

$1 + x^2 = t$

$$2x\,dx = dt\ ;\ x\,dx = \frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{2}\ arc\ tg\ x + \frac{x}{2(x^2+1)} + C}}$

152. RESOLUCIÓN

$$I = \int sec^3 x\,dx = \int sec\ x\cdot sec^2 x\,dx = sec\ x\ tg\ x -$$

$$-\int sec\ x\cdot tg^2 x\,dx = sec\ x\cdot tg\ x - \int sec\ x\cdot(sec^2 x - 1)\,dx =$$

$$= sec\ x\cdot tg\ x - \int sec^3 x\,dx + \int sec\ x\,dx = sec\ x\cdot tg\ x - I +$$

$$+ L\,|sec\ x + tg\ x|$$

$$2I = sec\ x\cdot tg\ x + L\,|sec\ x + tg\ x|$$

$$I = \frac{1}{2}[sec\ x\ tg\ x + L\,|sec\ x + tg\ x|] + C$$

CÁLCULOS AUXILIARES

$u = sec\ x\ ;\ du = sec\ x\ tg\ x\,dx$

$dv = sec^2 x\,dx\ ;\ v = \int sec^2 x\,dx = tg\ x$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{2}[sec\ x\cdot tg\ x + L\,|sec\ x + tg\ x|] + C}}$

153. RESOLUCIÓN

$$I = \int(x^2 - 2x + 1)\,Lx\,dx = \left(\frac{x^3}{3} - x^2 + x\right)Lx -$$

$$-\int\left(\frac{x^3}{3} - x^2 + x\right)\frac{dx}{x} = \left(\frac{x^3}{3} - x^2 + x\right)Lx -$$

$$-\frac{1}{3}\int x^2\,dx + \int x\,dx - \int dx =$$

$$= \left(\frac{x^3}{3} - x^2 + x\right)Lx - \frac{x^3}{9} + \frac{x^2}{2} - x + C$$

CÁLCULOS AUXILIARES

$$u = Lx\ ;\ du = \frac{dx}{x}$$

$$dv = (x^2 - 2x + 1)\,dx\ ;\ v = \frac{x^3}{3} - x^2 + x$$

SOLUCIÓN: $\boxed{\mathbf{I = \left(\frac{x^3}{3} - x^2 + x\right)Lx - \frac{x^3}{9} + \frac{x^2}{2} - x + C}}$

154. RESOLUCIÓN

$$I = \int (3x^2 - x + 5)\ sen\ x\ dx = -(3x^2 - x + 5)\ cos\ x +$$

$$+ \int (6x - 1)\ cos\ x\ dx = -(3x^2 - x + 5)\ cos\ x +$$

$$+ (6x - 1)\ sen\ x - 6 \int sen\ x\ dx =$$

$$= -(3x^2 - x + 5)\ cos\ x + (6x - 1)\ sen\ x + 6\ cos\ x + C$$

CÁLCULOS AUXILIARES

$u = 3x^2 - x + 5\ ;\ du = (6x - 1)\ dx$

$dv = sen\ x\ dx\ ;\ v = -cos\ x$

$u = 6x - 1\ ;\ u = 6$

$dv = cos\ x\ dx\ ;\ v = sen\ x$

SOLUCIÓN: $\boxed{\mathbf{I = -(3x^2 - x + 5)\ cos\ x + (6x - 1)\ sen\ x + 6\ cos\ x + C}}$

155. RESOLUCIÓN

$$I = \int x^3\ (Lx)^2\ dx = \frac{x^4}{4}\ (Lx)^2 - \int \frac{x^4}{4} \cdot \frac{2\ Lx}{x}\ dx =$$

$$= \frac{x^4}{4}\ (Lx)^2 - \frac{1}{2} \int x^3 \cdot Lx\ dx = \frac{x^4}{4}\ (Lx)^2 - \frac{1}{2} \left[\frac{x^4}{4} \cdot Lx - \right.$$

$$\left. - \frac{1}{4} \int x^3\ dx \right] = \frac{x^4}{4}\ (Lx)^2 - \frac{x^4}{8}\ Lx + \frac{x^4}{32} + C$$

CÁLCULOS AUXILIARES

$$u = (Lx)^2\ ;\ du = \frac{2\ Lx}{x}\ dx$$

$$dv = x^3\ dx\ ;\ v = \frac{x^4}{4}$$

$$u = Lx\ ;\ du = \frac{dx}{x}$$

$$dv = x^3\ dx\ ;\ v = \frac{x^4}{4}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^4}{4}\ (Lx)^2 - \frac{x^4}{8}\ Lx + \frac{x^4}{32} + C}}$

156. RESOLUCIÓN

$$I = \int L\ (x + 1)\ dx = x\ L\ (x + 1) - \int \frac{x\ dx}{x + 1} =$$

$$= x\ L\ (x + 1) - \int \left(1 - \frac{1}{x + 1}\right) dx =$$

$$= x\ L\ |x + 1| - \int dx + \int \frac{dx}{x + 1} =$$

$$= x\ L\ |x + 1| - x + L\ |x + 1| + C$$

CÁLCULOS AUXILIARES

$$u = L\ (x + 1)\ ;\ du = \frac{dx}{x + 1}$$

$$dv = dx\ ;\ v = \int dx = x$$

$$\begin{array}{r|l} x & x + 1 \\ \hline -x - 1 & 1 \\ \hline -1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = x\ L\ |x + 1| - x + L\ |x + 1| + C}}$

157. RESOLUCIÓN

$$I = \int \frac{x\ dx}{sen^2\ x} = -x\ ctg\ x + \int ctg\ x\ dx =$$

$$= -x\ ctg\ x + \int \frac{cos\ x}{sen\ x}\ dx = -\ x\ ctg\ x + \int \frac{dt}{t} =$$

$$= -x\ ctg\ x + L\ |t| + C = -x\ ctg\ x + L\ |sen\ x| + C$$

CÁLCULOS AUXILIARES

$u = x\ ;\ du = dx$

$$dv = \frac{dx}{sen^2\ x}\ ;\ v = \int \frac{dx}{sen^2\ x} = -ctg\ x$$

$sen\ x = t$

$cos\ x\ dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = -x\ ctg\ x + L\ |sen\ x| + C}}$

158. RESOLUCIÓN

$$I = \int sen\ Lx\ dx = x \cdot sen\ Lx - \int x \cdot \frac{1}{x} cos\ Lx\ dx =$$

$$= x\ sen\ Lx - \int cos\ Lx\ dx = x\ sen\ Lx -$$

$$- \left[x\ cos\ Lx + \int x \cdot \frac{1}{x} sen\ Lx\ dx\right] =$$

$$= x\ sen\ Lx - x\ cos\ Lx - \int sen\ Lx\ dx = x\ sen\ Lx - x\ cos\ Lx - I$$

$$2I = x\ (sen\ Lx - cos\ Lx)$$

$$I = \frac{x}{2}\ (sen\ Lx - cos\ Lx) + C$$

CÁLCULOS AUXILIARES

$$u = sen\ Lx\ ;\ du = \frac{1}{x} cos\ Lx \cdot dx$$

$$dv = dx\ ;\ v = x$$

$$u = cos\ Lx\ ;\ du = -\frac{1}{x} sen\ Lx\ dx$$

$$dv = dx\ ;\ v = x$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{2}\ (sen\ Lx - cos\ Lx) + C}}$

159. RESOLUCIÓN

$$I = \int \frac{Lx}{\sqrt{x}} dx = 2\sqrt{x} \cdot Lx - 2\int \frac{\sqrt{x}}{x} dx =$$

$$= 2\sqrt{x} \cdot Lx - 2\int \frac{\sqrt{x} \cdot \sqrt{x}}{x\sqrt{x}} dx = 2\sqrt{x} \cdot Lx - 2\int \frac{x\ dx}{x\sqrt{x}} =$$

$$= 2\sqrt{x} \cdot Lx - 2\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} \cdot Lx - 2 \cdot 2\sqrt{x} + C =$$

$$= 2\sqrt{x}\ (Lx - 2)\ + C$$

CÁLCULOS AUXILIARES

$$u = Lx\ ;\ du = \frac{dx}{x}$$

$$dv = \frac{dx}{\sqrt{x}}\ ;\ v = \int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{x}\ (Lx - 2) + C}}$

160. RESOLUCIÓN

$$I = \int e^{arc\ sen\ x}\ dx = x\ e^{arc\ sen\ x} - \int \frac{x \cdot e^{arc\ sen\ x}}{\sqrt{1 - x^2}} dx =$$

$$= x \cdot e^{arc\ sen\ x} - \left[-\sqrt{1 - x^2} \cdot e^{arc\ sen\ x} + \int e^{arc\ sen\ x}\ dx\right] =$$

$$= x \cdot e^{arc\ sen\ x} + \sqrt{1 - x^2} \cdot e^{arc\ sen\ x} - \int e^{arc\ sen\ x}\ dx =$$

$$= x\ e^{arc\ sen\ x} + \sqrt{1 - x^2} \cdot e^{arc\ senx} - I$$

$$2I = e^{arc\ sen\ x}\ (x + \sqrt{1 - x^2})$$

$$I = \frac{e^{arc\ sen\ x}}{2}\ (x + \sqrt{1 - x^2}) + C$$

CÁLCULOS AUXILIARES

$$u = e^{arc\ sen\ x}\ ;\ du = \frac{e^{arc\ sen\ x}}{\sqrt{1 - x^2}} dx$$

$$dv = dx\ ;\ v = x$$

$$u = e^{arc\ sen\ x}\ ;\ du = \frac{e^{arc\ sen\ x}}{\sqrt{1 - x^2}} dx$$

$$dv = \frac{x\ dx}{\sqrt{1 - x^2}}\ ;\ v = -\sqrt{1 - x^2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{e^{arc\ sen\ x}}{2}\ (x + \sqrt{1 - x^2}) + C}}$

161. RESOLUCIÓN

$$I = \int x \operatorname{arc\,tg} x^2\, dx = \frac{x^2}{2} \operatorname{arc\,tg} x^2 - \int \frac{x^2}{2} \cdot \frac{2x}{1+x^4}\, dx =$$

$$= \frac{x^2}{2} \operatorname{arc\,tg} x^2 - \int \frac{x^3}{1+x^4}\, dx = \frac{x^2}{2} \operatorname{arc\,tg} x^2 -$$

$$- \frac{1}{4} \int \frac{4x^3\, dx}{1+x^4} = \frac{x^2}{2} \operatorname{arc\,tg} x^2 - \frac{1}{4} L\,|1+x^4| + C$$

CÁLCULOS AUXILIARES

$$u = \operatorname{arc\,tg} x^2 \;;\; du = \frac{2x}{1+x^4}\, dx$$

$$dv = x\, dx \;;\; v = \int x\, dx = \frac{x^2}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^2}{2} \operatorname{arc\,tg} x^2 - \frac{1}{4} L\,|1+x^4| + C}}$

162. RESOLUCIÓN

$$I = \int \frac{x \operatorname{arc\,sen} x}{\sqrt{1-x^2}}\, dx = -\sqrt{1-x^2} \cdot \operatorname{arc\,sen} x + \int dx =$$

$$= -\sqrt{1-x^2} \cdot \operatorname{arc\,sen} x + x + C$$

CÁLCULOS AUXILIARES

$$u = \operatorname{arc\,sen} x \;;\; du = \frac{dx}{\sqrt{1-x^2}}$$

$$dv = \frac{x\, dx}{\sqrt{1-x^2}} \;;\; v = -\sqrt{1-x^2}$$

SOLUCIÓN: $\boxed{\mathbf{I = -\sqrt{1-x^2} \cdot \operatorname{arc\,sen} x + x + C}}$

163. RESOLUCIÓN

$$I = \int \frac{x\, e^x\, dx}{(1+x)^2} = e^x \left[L\,|1+x| + \frac{1}{1+x} \right] - \int \frac{x\, e^x\, dx}{(1+x)^2} =$$

$$= e^x \left[L\,|1+x| + \frac{1}{1+x} \right] - I$$

$$2I = e^x \left[L\,|1+x| + \frac{1}{1+x} \right]$$

$$I = \frac{e^x}{2} \left[L\,|1+x| + \frac{1}{1+x} \right] + C$$

CÁLCULOS AUXILIARES

$$u = e^x \;;\; du = e^x\, dx$$

$$dv = \frac{x\, dx}{(1+x)^2} \;;$$

$$v = \int \frac{x\, dx}{(1+x)^2} = \int \frac{(t-1)\, dt}{t^2} = \int \frac{dt}{t} - \int \frac{dt}{t^2} =$$

$$= L\,|t| + \frac{1}{t} = L\,|1+x| + \frac{1}{1+x}$$

$$1 + x = t \Rightarrow x = t - 1$$

$$dx = dt$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{e^x}{2} \left[L\,|1+x| + \frac{1}{1+x} \right] + C}}$

164. RESOLUCIÓN

$$I = \int \operatorname{arc\,cos} 2x\, dx = x \operatorname{arc\,cos} 2x - \int \frac{-2x\, dx}{\sqrt{1-4x^2}} =$$

$$= x \operatorname{arc\,cos} 2x - \frac{1}{2} \sqrt{1-4x^2} + C$$

CÁLCULOS AUXILIARES

$$u = \operatorname{arc\,cos} 2x \;;\; du = \frac{-2\, dx}{\sqrt{1-4x^2}}$$

$$dv = dx \;;\; v = x$$

$$\int \frac{-2x\, dx}{\sqrt{1-4x^2}} = \frac{1}{4} \int \frac{dt}{\sqrt{t}} = \frac{1}{4} \cdot 2\sqrt{t} = \frac{1}{2} \sqrt{t} =$$

$$= \frac{1}{2} \sqrt{1-4x^2}$$

$$1 - 4x^2 = t \Rightarrow -2x\, dx = \frac{dt}{4}$$

SOLUCIÓN: $\boxed{\mathbf{I = x \operatorname{arc\,cos} 2x - \frac{1}{2}\sqrt{1 - 4x^2} + C}}$

165. RESOLUCIÓN

$$I = \int \operatorname{arc\,tg} \sqrt{x}\, dx = \int \operatorname{arc\,tg}\, t \cdot 2t\, dt =$$

$$= 2\int t \operatorname{arc\,tg}\, t\, dt = 2\left[\frac{t^2}{2}\operatorname{arc\,tg}\, t - \frac{1}{2}\int \frac{t^2\, dt}{1 + t^2}\right] =$$

$$= t^2 \operatorname{arc\,tg}\, t - \int \frac{t^2\, dt}{1 + t^2} = t^2 \operatorname{arc\,tg}\, t - \int \frac{1 + t^2 - 1}{1 + t^2} dt =$$

$$= t^2 \operatorname{arc\,tg}\, t - \int \frac{1 + t^2}{1 + t^2} dt + \int \frac{dt}{1 + t^2} = t^2 \operatorname{arc\,tg}\, t - \int dt +$$

$$+ \int \frac{dt}{1 + t^2} = t^2 \operatorname{arc\,tg}\, t - t + \operatorname{arc\,tg}\, t + C =$$

$$= (t^2 + 1) \operatorname{arc\,tg}\, t - t + C = (x + 1) \operatorname{arc\,tg} \sqrt{x} - \sqrt{x} + C$$

CÁLCULOS AUXILIARES

$x = t^2 \;;\; dx = 2t\, dt$

$u = \operatorname{arc\,tg}\, t \;;\; du = \dfrac{dt}{1 + t^2}$

$dv = t\, dt \;;\; v = \displaystyle\int t\, dt = \frac{t^2}{2}$

SOLUCIÓN: $\boxed{\mathbf{I = (x + 1) \operatorname{arc\,tg} \sqrt{x} - \sqrt{x} + C}}$

166. RESOLUCIÓN

$$I = \int \frac{L\,(x + 1)}{\sqrt{x + 1}} dx = 2\sqrt{x + 1} \cdot L\,(x + 1) - 2\int \frac{\sqrt{x + 1}}{x + 1} dx =$$

$$= 2\sqrt{x + 1} \cdot L\,(x + 1) - 2\int \frac{\sqrt{x + 1} \cdot \sqrt{x + 1}}{(x + 1)\sqrt{x + 1}} dx =$$

$$= 2\sqrt{x + 1} \cdot L\,(x + 1) - 2\int \frac{dx}{\sqrt{x + 1}} =$$

$$= 2\sqrt{x + 1} \cdot L\,(x + 1) - 4\sqrt{x + 1} + C$$

CÁLCULOS AUXILIARES

$u = L\,(x + 1) \;;\; du = \dfrac{dx}{x + 1}$

$dv = \dfrac{dx}{\sqrt{x + 1}} \;;\; v = \displaystyle\int \frac{2\, dx}{2\sqrt{x + 1}} = 2\sqrt{x + 1}$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{x + 1}\,[L\,(x + 1) - 2] + C}}$

167. RESOLUCIÓN

$$I = \int x^2 \cdot \operatorname{arc\,sen}\, x\, dx = \frac{x^3}{3}\operatorname{arc\,sen}\, x - \frac{1}{3}\int \frac{x^3\, dx}{\sqrt{1 - x^2}} =$$

$$= \frac{x^3}{3}\operatorname{arc\,sen}\, x - \frac{1}{3}\int \frac{x^2 \cdot x\, dx}{\sqrt{1 - x^2}} =$$

$$= \frac{x^3}{3}\operatorname{arc\,sen}\, x - \frac{1}{3}\int \frac{(1 - t^2)(-t)\, dt}{t} =$$

$$= \frac{x^3}{3}\operatorname{arc\,sen}\, x + \frac{1}{3}\int (1 - t^2)\, dt =$$

$$= \frac{x^3}{3}\operatorname{arc\,sen}\, x + \frac{1}{3}\int dt - \frac{1}{3}\int t^2\, dt =$$

$$= \frac{x^3}{3}\operatorname{arc\,sen}\, x + \frac{1}{3}t - \frac{1}{3} \cdot \frac{t^3}{3} + C =$$

$$= \frac{x^3}{3}\operatorname{arc\,sen}\, x + \frac{\sqrt{1 - x^2}}{3} - \frac{(1 - x^2)\sqrt{1 - x^2}}{9} + C$$

CÁLCULOS AUXILIARES

$u = \operatorname{arc\,sen}\, x \;;\; du = \dfrac{dx}{\sqrt{1 - x^2}}$

$dv = x^2\, dx \;;\; v = \displaystyle\int x^2\, dx = \frac{x^3}{3}$

$1 - x^2 = t^2 \Rightarrow x^2 = 1 - t^2$

$2x\, dx = -2t\, dt$

$x\, dx = -t\, dt$

SOLUCIÓN:

$$\boxed{\mathbf{I = \frac{x^3}{3}\ arc\ sen\ x + \frac{\sqrt{1-x^2}}{3} - \frac{(1-x^2)\sqrt{1-x^2}}{9} + C}}$$

168. RESOLUCIÓN

$$I = \int x\,[L\,(1+x^2) + e^{-x}]\,dx = \int x\,L\,(1+x^2)\,dx + \int x\,e^{-x}\,dx =$$

$$= I_1 + I_2 = -\frac{x^2}{2} + \frac{x^2+1}{2} L\,(1+x^2) - e^{-x}\,(x+1) + C$$

CÁLCULOS AUXILIARES

$$u = L\,(1+x^2)\ ;\ du = \frac{2x\,dx}{1+x^2}$$

$$dv = x\,dx\ ;\ v = \int x\,dx = \frac{x^2}{2}$$

$$\begin{array}{l|l} x^3 & x^2+1 \\ \hline -x^3 - x & x \\ \hline -x & \end{array}$$

$$u = x\ ;\ du = dx$$

$$dv = e^{-x}\,dx\ ;\ v = -e^{-x}$$

$$I_1 = \int x\,L\,(1+x^2)\,dx = \frac{x^2}{2} L\,(1+x^2) - \int \frac{x^2}{2} \cdot \frac{2x\,dx}{1+x^2} =$$

$$= \frac{x^2}{2} L\,(1+x^2) - \int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} L\,(1+x^2) -$$

$$- \int \left(x - \frac{x}{1+x^2}\right) dx = \frac{x^2}{2} L\,(1+x^2) - \int x\,dx +$$

$$+ \frac{1}{2}\int \frac{2x\,dx}{1+x^2} = \frac{x^2}{2} L\,(1+x^2) - \frac{x^2}{2} + \frac{1}{2} L\,(1+x^2) + C_1$$

$$I_2 = \int x\,e^{-x}\,dx = -x\,e^{-x} + \int e^{-x}\,dx = -x\,e^{-x} - e^{-x} + C_2 =$$

$$= -x\,(e^{-x} + 1) + C_2$$

SOLUCIÓN:

$$\boxed{\mathbf{I = -\frac{x^2}{2} + \frac{x^2+1}{2} L\,(1+x^2) - e^{-x}\,(x+1) + C}}$$

169. RESOLUCIÓN

$$I = \int \frac{dx}{(x-1)(x+1)} = \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} =$$

$$= \frac{1}{2} L\,|x-1| - \frac{1}{2} L\,|x+1| + C = L\left|\frac{\sqrt{x-1}}{\sqrt{x+1}}\right| + C$$

$$\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A\,(x+1) + B\,(x-1)}{(x-1)(x+1)}$$

$$1 = A\,(x+1) + B\,(x-1) = (A+B)\,x + A - B$$

$$\left.\begin{array}{l} 0 = A + B \\ 1 = A - B \end{array}\right\} \Rightarrow A = \frac{1}{2}\ ;\ B = -\frac{1}{2}$$

SOLUCIÓN:

$$\boxed{\mathbf{I = L\left|\frac{\sqrt{x-1}}{\sqrt{x+1}}\right| + C}}$$

170. RESOLUCIÓN

$$I = \int \frac{x\,dx}{x^2 - x - 2} = \frac{2}{3}\int \frac{dx}{x-2} + \frac{1}{3}\int \frac{dx}{x+1} =$$

$$= \frac{2}{3} L\,|x-2| + \frac{1}{3} L\,|x+1| + C =$$

$$= L\,|\sqrt[3]{(x-2)^2}| + L\,|\sqrt[3]{x+1}| + C =$$

$$= L\,|\sqrt[3]{(x-2)^2\,(x+1)}| + C$$

CÁLCULOS AUXILIARES

$$x^2 - x - 2 = 0 \Rightarrow x_1 = 2\ ;\ x_2 = -1$$

$$x^2 - x - 2 = (x-2)\,(x+1)$$

$$\frac{x}{x^2-x-2}=\frac{x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}=$$

$$=\frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$x=A(x+1)+B(x-2)$$

para $x=2 \Rightarrow 2=3A \Rightarrow A=\frac{2}{3}$

para $x=-1 \Rightarrow -1=-3B \Rightarrow B=\frac{1}{3}$

SOLUCIÓN: $\boxed{\mathbf{I=L\left|\sqrt[3]{(x-2)^2(x+1)}\right|+C}}$

171. RESOLUCIÓN

$$I=\int\frac{dx}{x^2-9}=\frac{1}{6}\int\frac{dx}{x-3}-\frac{1}{6}\int\frac{dx}{x+3}=$$

$$=\frac{1}{6}L|x-3|-\frac{1}{6}L|x+3|+C=L\left|\sqrt[6]{\frac{x-3}{x+3}}\right|+C$$

$$\frac{1}{x^2-9}=\frac{1}{(x-3)(x+3)}=\frac{A}{x-3}+\frac{B}{x+3}$$

$$1=A(x+3)+B(x-3)$$

para $x=3 \Rightarrow 1=6A \Rightarrow A=\frac{1}{6}$

para $x=-3 \Rightarrow 1=-6B \Rightarrow B=-\frac{1}{6}$

SOLUCIÓN: $\boxed{\mathbf{I=L\left|\sqrt[6]{\frac{x-3}{x+3}}\right|+C}}$

172. RESOLUCIÓN

$$I=\int\frac{dx}{x(x-1)(x-2)}=\frac{1}{2}\int\frac{dx}{x}-\int\frac{dx}{x-1}+\frac{1}{2}\int\frac{dx}{x-2}=$$

$$=\frac{1}{2}L|x|-L|x-1|+\frac{1}{2}L|x-2|+C=L\left|\frac{\sqrt{x(x-2)}}{x-1}\right|+C$$

$$\frac{1}{x(x-1)(x-2)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x-2}$$

$$1=A(x-1)(x-2)+Bx(x-2)+Cx(x-1)$$

para $x=0 \Rightarrow 1=2A \Rightarrow A=\frac{1}{2}$

para $x=1 \Rightarrow 1=-B \Rightarrow B=-1$

para $x=2 \Rightarrow 1=2C \Rightarrow C=\frac{1}{2}$

SOLUCIÓN: $\boxed{\mathbf{I=L\left|\frac{\sqrt{x(x-2)}}{x-1}\right|+C}}$

173. RESOLUCIÓN

$$I=\int\frac{x^3\,dx}{x^2-x-2}=\int\left(x+1+\frac{3x+2}{x^2-x-2}\right)dx=$$

$$=\int x\,dx+\int dx+\int\frac{3x+2}{x^2-x-2}dx=\frac{x^2}{2}+x+$$

$$+\frac{8}{3}\int\frac{dx}{x-2}+\frac{1}{3}\int\frac{dx}{x+1}=\frac{x^2}{2}+x+\frac{8}{3}L|x-2|+$$

$$+\frac{1}{3}L|x-1|+C$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} x^3 & x^2-x-2 \\ \hline -x^3+x^2+2x & x+1 \\ \hline x^2+2x & \\ -x^2+x+2 & \\ \hline 3x+2 & \end{array}$$

$$\frac{3x+2}{x^2-x-2}=\frac{3x+2}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$$

$$3x+2=A(x+1)+B(x-2)$$

$$3x+2=(A+B)x+A-2B$$

$$\left.\begin{array}{l}3=A+B\\2=A-2B\end{array}\right\}\Rightarrow A=\frac{8}{3}\ ;\ B=\frac{1}{3}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^2}{2} + x + \frac{8}{3} L\,|x-2| + \frac{1}{3} L\,|x-1| + C}}$

174. RESOLUCIÓN

$$I = \int \frac{(4x-2)\,dx}{x^3 - x^2 - 2x} = \int \frac{dx}{x} + \int \frac{dx}{x-2} - 2\int \frac{dx}{x+1} =$$

$$= L\,|x| + L\,|x-2| - 2L\,|x+1| + C = L\left|\frac{x\,(x-2)}{x+1}\right| + C$$

CÁLCULOS AUXILIARES

$x^3 - x^2 - 2x = 0$

$x\,(x^2 - x - 2) = 0$

$x = 0$

$x^2 - x - 2 = 0 \Rightarrow x = 2 \;;\; x = -1$

$x^3 - x^2 - 2x = x\,(x-2)\,(x+1)$

$$\frac{4x-2}{x^3 - x^2 - 2x} = \frac{4x-2}{x\,(x-2)\,(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

$4x - 2 = A\,(x-2)\,(x+1) + Bx\,(x+1) + Cx\,(x-2)$

para $x = 0 \Rightarrow -2 = -2A \Rightarrow A = 1$

para $x = -1 \Rightarrow -6 = 3C \Rightarrow C = -2$

para $x = 2 \Rightarrow 6 = 6B \Rightarrow B = 1$.

SOLUCIÓN: $\boxed{\mathbf{I = L\left|\frac{x\,(x-2)}{x+1}\right| + C}}$

175. RESOLUCIÓN

$$I = \int \frac{(5x^2 - 3)\,dx}{x^3 - x} = 3\int \frac{dx}{x} + \int \frac{dx}{x+1} - \int \frac{dx}{x-1} =$$

$$= 3\,L\,|x| + L\,|x+1| + L\,|x-1| + C = L\,|x^3\,(x^2-1)| + C$$

CÁLCULOS AUXILIARES

$x^3 - x = 0$

$x\,(x^2 - 1) = 0$

$x = 0$

$x^2 - 1 = 0 \Rightarrow x_1 = -1 \;;\; x = 1$

$x^3 - x = x\,(x+1)\,(x-1)$

$$\frac{5x^2 - 3}{x^3 - x} = \frac{5x^2 - 3}{x\,(x+1)\,(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$5x^2 - 3 = A\,(x+1)\,(x-1) + Bx\,(x-1) + Cx\,(x+1)$

para $x = 0 \Rightarrow -3 = -A \Rightarrow A = 3$

para $x = -1 \Rightarrow 2 = 2B \Rightarrow B = 1$

para $x = 1 \Rightarrow 2 = 2C \Rightarrow C = 1$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|x^3\,(x^2-1)| + C}}$

176. RESOLUCIÓN

$$I = \int \frac{(4x^3 + 2x^2 + 1)}{4x^3 - x}\,dx = \int \left(1 + \frac{2x^2 + x + 1}{4x^3 - x}\right) dx =$$

$$= \int dx + \int \frac{2x^2 + x + 1}{4x^3 - x}\,dx = \int dx - \int \frac{dx}{x} + \int \frac{dx}{2x+1} +$$

$$+ 2\int \frac{dx}{2x-1} = x - L\,|x| + \frac{1}{2} L\,|2x+1| + L\,|2x-1| + C =$$

$$= x + L\left|\frac{(2x-1)\sqrt{2x+1}}{x}\right| + C$$

$$\frac{2x^2 + x + 1}{4x^3 - x} = \frac{2x^2 + x + 1}{x\,(4x^2 - 1)} = \frac{2x^2 + x + 1}{x\,(2x+1)\,(2x-1)} =$$

$$= \frac{A}{x} + \frac{B}{2x+1} + \frac{C}{2x-1}$$

$2x^2 + x + 1 = A\,(2x+1)\,(2x-1) + Bx\,(2x-1) + Cx\,(2x+1)$

$2x^2 + x + 1 = (4A + 2B + 2C)\,x^2 + (-B + C)\,x - A$

$$\left.\begin{array}{l} 2 = 4A + 2B + 2C \\ 1 = -B + C \\ 1 = -A \end{array}\right\} \Rightarrow A = -1 \;;\; B = 1 \;;\; C = 2$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} 4x^3 + 2x^2 \quad + 1 & 4x^3 - x \\ -4x^3 \quad + x & 1 \\ \hline 2x^2 + x + 1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = x + L\left|\dfrac{(2x-1)\sqrt{2x+1}}{x}\right| + C}}$

177. RESOLUCIÓN

$$I = \int \frac{(3x^2 + 5x)}{(x-1)(x+1)^2}\,dx = 2\int \frac{dx}{x-1} + \int \frac{dx}{(x+1)^2} + \int \frac{dx}{x+1} =$$

$$= 2L|x-1| - \frac{1}{x+1} + L|x+1| + C = -\frac{1}{x+1} +$$

$$+ L|(x-1)^2(x+1)| + C$$

$$\frac{3x^2 + 5x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1}$$

$$3x^2 + 5x = A(x+1)^2 + B(x-1) + C(x-1)(x+1)$$

$$3x^2 + 5x = (A + C)x^2 + (2A + B)x + (A - B - C)$$

$$\left.\begin{array}{l} 3 = A + C \\ 5 = 2A + B \\ 0 = A - B - C \end{array}\right\} \Rightarrow A = 2\ ;\ B = 1\ ;\ C = 1$$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{x+1} + L\,|(x-1)^2(x+1)| + C}}$

178. RESOLUCIÓN

$$I = \int \frac{x^2\,dx}{(x+1)^3} = \int \frac{dx}{(x+1)^3} - 2\int \frac{dx}{(x+1)^2} + \int \frac{dx}{x+1} =$$

$$= -\frac{1}{2(x+1)^2} + \frac{2}{x+1} + L|x+1| + C$$

$$\frac{x^2}{(x+1)^3} = \frac{A}{(x+1)^3} + \frac{B}{(x+1)^2} + \frac{C}{x+1}$$

$$x^2 = A + B(x+1) + C(x+1)^2$$

$$x^2 = Cx^2 + (B + 2C)x + A + B + C$$

$$\left.\begin{array}{l} 1 = C \\ 0 = A + 2C \\ 0 = A + B + C \end{array}\right\} \Rightarrow A = 1\ ;\ B = -2\ ;\ C = 1$$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{2(x+1)^2} + \dfrac{2}{x+1} + L\,|x+1| + C}}$

179. RESOLUCIÓN

$$I = \int \frac{dx}{(1+x)(1-x^2)} = \frac{1}{2}\int \frac{dx}{(1+x)^2} + \frac{1}{4}\int \frac{dx}{1+x} +$$

$$+ \frac{1}{4}\int \frac{dx}{1-x} = -\frac{1}{2(1+x)} + \frac{1}{4}L|1+x| -$$

$$-\frac{1}{4}L|1-x| + C = -\frac{1}{2(1+x)} + L\left|\sqrt[4]{\frac{1+x}{1-x}}\right| + C$$

CÁLCULOS AUXILIARES

$$\frac{1}{(1+x)(1-x^2)} = \frac{1}{(1+x)(1+x)(1-x)} =$$

$$= \frac{1}{(1+x)^2(1-x)} = \frac{A}{(1+x)^2} + \frac{B}{1+x} + \frac{C}{1-x}$$

$$1 = A(1-x) + B(1+x)(1-x) + C(1+x)^2$$

$$1 = (-B + C)x^2 + (-A + 2C)x + A + B + C$$

$$\left.\begin{array}{l} 0 = -B + C \\ 0 = -A + 2C \\ 1 = A + B + C \end{array}\right\} \Rightarrow A = \frac{1}{2}\ ;\ B = \frac{1}{4}\ ;\ C = \frac{1}{4}$$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{2(1+x)} + L\left|\sqrt[4]{\dfrac{1+x}{1-x}}\right| + C}}$

180. RESOLUCIÓN

$$I = \int \frac{(2x+3)}{x^3+x^2-2x}\,dx = -\frac{3}{2}\int \frac{dx}{x} + \frac{5}{3}\int \frac{dx}{x-1} -$$

$$-\frac{1}{6}\int \frac{dx}{x+2} = -\frac{3}{2}L\,x + \frac{5}{3}L\,|x-1| - \frac{1}{6}L\,|x+2| + C =$$

$$= L\left|\frac{(x-1)^{5/3}}{x^{3/2}\cdot(x+2)^{1/6}}\right| + C$$

CÁLCULOS AUXILIARES

$$\frac{2x+3}{x^3+x^2-2x} = \frac{2x+3}{x\,(x-1)\,(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

$$2x+3 = A\,(x-1)\,(x+2) + Bx\,(x+2) + Cx\,(x-1)$$

$$\text{para } x=0 \Rightarrow 3 = -2A \;;\; A = -\frac{3}{2}$$

$$\text{para } x=1 \Rightarrow 5 = 3B \;;\; B = \frac{5}{3}$$

$$\text{para } x=-2 \Rightarrow 1 = -6C \;;\; C = -\frac{1}{6}$$

SOLUCIÓN: $\boxed{\mathbf{I = L\left|\dfrac{(x-1)^{5/3}}{x^{3/2}\,(x+2)^{1/6}}\right| + C}}$

181. RESOLUCIÓN

$$I = \int \frac{(x^3+1)\,dx}{x\,(x-1)^3} = -\int \frac{dx}{x} + 2\int \frac{dx}{(x-1)^3} + \int \frac{dx}{(x-1)^2} +$$

$$+ 2\int \frac{dx}{x-1} = -L|x| + \frac{2\,(x-1)^{-2}}{-2} + \frac{(x-1)^{-1}}{-1} +$$

$$+ 2\,L\,|x-1| + C = -L|x| - \frac{1}{(x-1)^2} - \frac{1}{(x-1)} +$$

$$+ L\,|(x-1)^2| + C = L\left|\frac{(x-1)^2}{x}\right| - \frac{x}{(x-1)^2} + C$$

CÁLCULOS AUXILIARES

$$\frac{x^3+1}{x\,(x-1)^3} = \frac{A}{x} + \frac{B}{(x-1)^3} + \frac{C}{(x-1)^2} + \frac{D}{x-1}$$

$$x^3+1 = A\,(x-1)^3 + Bx + Cx\,(x-1) + Dx\,(x-1)^2$$

$$\left.\begin{array}{l}\text{para } x=0 \Rightarrow A = -1\\ \text{para } x=1 \Rightarrow B = 2\\ \text{para } x=2 \Rightarrow 9 = 3 + 2C + 2D\\ \text{para } x=-1 \Rightarrow 0 = 6 + 2C - 4D\end{array}\right\} \Rightarrow \begin{array}{l}C = 1\\ D = 2\end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = L\left|\dfrac{(x-1)^2}{x}\right| - \dfrac{x}{(x-1)^2} + C}}$

182. RESOLUCIÓN

$$I = \int \frac{x^2\,dx}{(x-1)^3} = \int \frac{dx}{(x-1)^3} + 2\int \frac{dx}{(x-1)^2} + \int \frac{dx}{x-1} =$$

$$= \int (x-1)^{-3}\,dx + 2\int (x-1)^{-2}\,dx + \int \frac{dx}{x-1} =$$

$$= -\frac{1}{2\,(x-1)^2} - \frac{2}{x-1} + L\,|x-1| + C$$

CÁLCULOS AUXILIARES

$$\frac{x^2}{(x-1)^3} = \frac{A}{(x-1)^3} + \frac{B}{(x-1)^2} + \frac{C}{x-1}$$

$$x^2 = A + B\,(x-1) + C\,(x-1)^2$$

$$x^2 = Cx^2 + (B-2C)\,x + A - B + C$$

$$\left.\begin{array}{l}1 = C\\ 0 = B - 2C\\ 0 = A - B + C\end{array}\right\} \Rightarrow A = 1 \;;\; B = 2 \;;\; C = 1$$

SOLUCIÓN: $\boxed{\mathbf{I = -\dfrac{1}{2\,(x-1)^2} - \dfrac{2}{x-1} + L\,|x-1| + C}}$

183. RESOLUCIÓN

$$I = \int \frac{(x^4 - 8)\,dx}{x^3 + 2x^2} = \int \left(x - 2 + \frac{4x^2 - 8}{x^3 + 2x^2} \right) dx =$$

$$= \int x\,dx - 2\int dx - 4\int \frac{dx}{x^2} + 2\int \frac{dx}{x} + 2\int \frac{dx}{x+2} =$$

$$= \frac{x^2}{2} - 2x + \frac{4}{x} + 2L\,|x| + 2\,L\,|x+2| + C =$$

$$= \frac{x^2}{2} - 2x + \frac{4}{x} + L\,|x^2\,(x+2)^2| + C$$

CÁLCULOS AUXILIARES

$$\begin{array}{r|l} x^4 - 8 & x^3 + 2x^2 \\ \hline -x^4 - 2x^3 & x - 2 \\ \hline -2x^3 & \\ 2x^3 + 4x^2 & \\ \hline 4x^2 - 8 & \end{array}$$

$$\frac{4x^2 - 8}{x^3 + 2x^2} = \frac{4x^2 - 8}{x^2\,(x+2)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+2}$$

$$4x^2 - 8 = A\,(x+2) + Bx\,(x+2) + Cx^2$$

$$4x^2 - 8 = (B + C)\,x^2 + (A + 2B)\,x + 2A$$

$$\left.\begin{array}{l} 4 = B + C \\ 0 = A + 2B \\ -8 = 2A \end{array}\right\} \Rightarrow A = -4\ ;\ B = 2\ ;\ C = 2$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x^2}{2} - 2x + \frac{4}{x} + L\,|x^2\,(x+2)^2| + C}}$

184. RESOLUCIÓN

$$I = \int \frac{8\,dx}{x^3 - 4x} = -2\int \frac{dx}{x} + \int \frac{dx}{x+2} + \int \frac{dx}{x-2} =$$

$$= -2\,L\,|x| + L\,|x+2| + L\,|x-2| + C =$$

$$= L\left| \frac{(x+2)\,(x-2)}{x^2} \right| + C = L\left| \frac{x^2 - 4}{x^2} \right| + C$$

CÁLCULOS AUXILIARES

$$\frac{8}{x^3 - 4x} = \frac{8}{x\,(x^2 - 4)} = \frac{8}{x\,(x+2)\,(x-2)} =$$

$$= \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$8 = A\,(x+2)\,(x-2) + Bx\,(x-2) + Cx\,(x+2)$$

para $x = 0 \Rightarrow 8 = -4A \Rightarrow A = -2$

para $x = -2 \Rightarrow 8 = 8B \Rightarrow B = 1$

para $x = 2 \Rightarrow 8 = 8C \Rightarrow C = 1$

SOLUCIÓN: $\boxed{\mathbf{I = L\left| \frac{x^2 - 4}{x^2} \right| + C}}$

185. RESOLUCIÓN

$$I = \int \frac{(3x^2 + 11x + 2)}{(x+3)\,(x^2 - 1)}\,dx = -\frac{1}{2}\int \frac{dx}{x+3} + \frac{3}{2}\int \frac{dx}{x+1} +$$

$$+ 2\int \frac{dx}{x-1} = -\frac{1}{2}L\,|x+3| + \frac{3}{2}L\,|x+1| + 2\,L\,|x-1| + C =$$

$$= L\left| \frac{\sqrt{(x+1)^3} \cdot (x-1)^2}{\sqrt{x+3}} \right| + C$$

CÁLCULOS AUXILIARES

$$\frac{3x^2 + 11x + 2}{(x+3)\,(x^2 - 1)} = \frac{3x^2 + 11x + 2}{(x+3)\,(x+1)\,(x-1)} =$$

$$= \frac{A}{x+3} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$3x^2 + 11x + 2 = A\,(x+1)\,(x-1) + B\,(x+3)\,(x-1) +$$

$$+ C\,(x+3)\,(x+1)$$

para $x = -3 \Rightarrow -4 = 8A \Rightarrow A = -\frac{1}{2}$

para $x = -1 \Rightarrow -6 = -4B \Rightarrow B = \frac{3}{2}$

para $x = 1 \Rightarrow 16 = 8C \Rightarrow C = 2$

SOLUCIÓN: $\boxed{\mathbf{I = L\left| \frac{(x-1)^2 \cdot \sqrt{(x+1)^3}}{\sqrt{x+3}} \right| + C}}$

186. RESOLUCIÓN

$$= \int \frac{dx}{x^2 + 4x + 5} = \int \frac{dx}{(x+2)^2 + 1} = \text{arc tg } (x+2) + C$$

CÁLCULOS AUXILIARES

$x^2 + 4x + 5 = 0 \; ; \; x_1 = -2 + i \; ; \; x_2 = -2 - i$

$x^2 + 4x + 5 = (x + 2 - i)\,(x + 2 + i) =$

$= [(x+2) - i]\,[(x+2) + i] = (x+2)^2 - i^2 = (x+2)^2 + 1$

SOLUCIÓN: $\boxed{\mathbf{I = arc\ tg\ (x + 2) + C}}$

187. RESOLUCIÓN

$$I = \int \frac{4x - 5}{x^2 - 4x + 20}\,dx = \int \frac{4x - 5 + 8 - 8}{(x-2)^2 + 4^2}\,dx =$$

$$= \int \frac{4\,(x-2)\,dx}{(x-2)^2 + 4} + 3\int \frac{dx}{(x-2)^2 + 4^2} =$$

$$= 2\int \frac{2\,(x-2)\,dx}{(x-2)^2 + 4^2} + 3\int \frac{dx}{(x-2)^2 + 4^2} =$$

$$= 2\int \frac{dt}{t} + 3\int \frac{4\,dz}{4^2 z^2 + 4^2} = 2\,L\,|t| + \frac{3}{4}\int \frac{dz}{z^2 + 1} =$$

$$= 2\,L\,|t| + \frac{3}{4}\,\text{arc tg } z + C =$$

$$= 2\,L\,|(x-2)^2 + 4^2| + \frac{3}{4}\,\text{arc tg } \frac{x-2}{4} + C$$

CÁLCULOS AUXILIARES

$x^2 - 4x + 20 = (x-2)^2 + 4^2$

$(x-2)^2 + 4^2 = t \qquad x - 2 = 4z \; ; \; z = \dfrac{x-2}{4}$

$2\,(x-2)\,dx = dt \qquad dx = 4\,dz$

SOLUCIÓN: $\boxed{\mathbf{I = 2L\,|(x-2)^2 + 4^2| + \frac{3}{4}\,arc\ tg\ \frac{x-2}{4} + C}}$

188. RESOLUCIÓN

$$I = \int \frac{dx}{x\,(1 + x^2)} = \int \frac{dx}{x} - \int \frac{x\,dx}{1 + x^2} =$$

$$= \int \frac{dx}{x} - \frac{1}{2}\int \frac{2x\,dx}{1 + x^2} = L\,|x| - \frac{1}{2}L\,|1 + x^2| + C =$$

$$= L\left|\frac{x}{\sqrt{1 + x^2}}\right| + C$$

CÁLCULOS AUXILIARES

$$\frac{1}{x\,(1 + x^2)} = \frac{A}{x} + \frac{Bx + C}{1 + x^2} =$$

$$= \frac{A\,(1 + x^2) + (Bx + C)\,x}{x\,(1 + x^2)}$$

$1 = A\,(1 + x^2) + (Bx + C)\,x = (A + B)\,x^2 + Cx + A$

$$\left.\begin{array}{l} 0 = A + B \\ 0 = C \\ 1 = A \end{array}\right\} \Rightarrow A = 1 \; ; \; B = -1 \; ; \; C = 0$$

SOLUCIÓN: $\boxed{\mathbf{I = L\left|\frac{x}{\sqrt{1 + x^2}}\right| + C}}$

189. RESOLUCIÓN

$$I = \int \frac{dx}{3x^2 - 6x + 9} = \frac{1}{3}\int \frac{dx}{x^2 - 2x + 3} =$$

$$= \frac{1}{3}\int \frac{dx}{(x-1)^2 + 2} = \frac{1}{6}\int \frac{dx}{\frac{(x-1)^2}{2} + 1} =$$

$$= \frac{1}{6}\int \frac{dx}{\left(\frac{x-1}{\sqrt{2}}\right)^2 + 1} = \frac{1}{6}\int \frac{\sqrt{2}\,dt}{t^2 + 1} =$$

$$= \frac{\sqrt{2}}{6} \text{ arc tg } t + C = \frac{\sqrt{2}}{6} \text{ arc tg } \frac{x-1}{\sqrt{2}} + C$$

CÁLCULOS AUXILIARES

$$x^2 - 2x + 3 = (x-1)^2 + (\sqrt{2})^2$$

$$\frac{x-1}{\sqrt{2}} = t$$

$$\frac{dx}{\sqrt{2}} = dt \; ; \; dx = \sqrt{2}\, dt$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{\sqrt{2}}{6} \text{ arc tg } \frac{x-1}{\sqrt{2}} + C}}$

190. RESOLUCIÓN

$$I = \int \frac{4\,dx}{x^3 + 4x} = \int \frac{dx}{x} - \int \frac{x\,dx}{x^2 + 4} =$$

$$= \int \frac{dx}{x} - \frac{1}{2}\int \frac{2x\,dx}{x^2 + 4} = L\,|x| - \frac{1}{2} L\,|x^2 + 4| + C =$$

$$= L \left| \frac{x}{\sqrt{x^2 + 4}} \right| + C$$

CÁLCULOS AUXILIARES

$$\frac{4}{x^3 + 4x} = \frac{4}{x\,(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

$$4 = A\,(x^2 + 4) + (Bx + C)\,x$$

$$4 = (A + B)\,x^2 + Cx + 4A$$

$$\left.\begin{array}{l} 0 = A + B \\ 0 = C \\ 4 = 4A \end{array}\right\} \Rightarrow A = 1 \; ; \; B = -1 \; ; \; C = 0$$

SOLUCIÓN: $\boxed{\mathbf{I = L \left| \frac{x}{\sqrt{x^2 + 4}} \right| + C}}$

191. RESOLUCIÓN

$$I = \int \frac{(4x^2 + 6)\,dx}{x^3 + 3x} = 2\int \frac{dx}{x} + \int \frac{2x\,dx}{x^2 + 3} =$$

$$= 2\,Lx + L\,|x^2 + 3| + C = L\,|x^2\,(x^2 + 3)| + C$$

CÁLCULOS AUXILIARES

$$\frac{4x^2 + 6}{x^3 + 3x} = \frac{4x^2 + 6}{x\,(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$

$$4x^2 + 6 = A\,(x^2 + 3) + (Bx + C)\,x$$

$$4x^2 + 6 = (A + B)\,x^2 + Cx + 3A$$

$$\left.\begin{array}{l} 4 = A + B \\ 0 = C \\ 6 = 3A \end{array}\right\} \Rightarrow A = 2 \; ; \; B = 2 \; ; \; C = 0$$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|x^2\,(x^2 + 3)| + C}}$

192. RESOLUCIÓN

$$I = \int \frac{(x^2 + x)\,dx}{(x-1)\,(x^2 + 1)} = \int \frac{dx}{x-1} + \int \frac{dx}{x^2 + 1} =$$

$$= L\,|x - 1| + \text{arc tg } x + C$$

CÁLCULOS AUXILIARES

$$\frac{x^2 + x}{(x-1)\,(x^2 + 1)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + 1} =$$

$$= \frac{A\,(x^2 + 1) + (Bx + C)\,(x-1)}{(x-1)\,(x^2 + 1)}$$

$$x^2 + x = A\,(x^2 + 1) + (Bx + C)\,(x - 1)$$

$$x^2 + x = (A + B)\,x^2 + (C - B)\,x + A - C$$

$$\left.\begin{array}{l} 1 = A + B \\ 1 = C - B \\ 0 = A - C \end{array}\right\} \Rightarrow A = 1 \; ; \; B = 0 \; ; \; C = 1$$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|x - 1| + \text{arc tg } x + C}}$

193. RESOLUCIÓN

$$I = \int \frac{(x-18)\,dx}{4x^3+9x} = -2\int \frac{dx}{x} + \int \frac{(8x+1)\,dx}{4x^2+9} =$$

$$= -2\,L\,|x| + \int \frac{8x\,dx}{4x^2+9} + \int \frac{dx}{4x^2+9} =$$

$$= -2\,L\,|x| + L\,|4x^2+9| + \int \frac{\frac{dx}{9}}{\left(\frac{2x}{3}\right)^2+1} = -2\,L\,|x| +$$

$$+ L\,|4x^2+9| + \frac{1}{6}\,arc\,tg\,\frac{2x}{3} + C =$$

$$= L\left|\frac{4x^2+9}{x^2}\right| + \frac{1}{6}\,arc\,tg\,\frac{2x}{3} + C$$

CÁLCULOS AUXILIARES

$$\frac{x-18}{4x^3+9x} = \frac{x-18}{x\,(4x^2+9)} = \frac{A}{x} + \frac{Bx+C}{4x^2+9}$$

$$x - 18 = A\,(4x^2+9) + (Bx+C)\,x$$

$$x - 18 = (4A+B)\,x^2 + Cx + 9A$$

$$\left.\begin{array}{l} 0 = 4A + B \\ 1 = C \\ -18 = 9A \end{array}\right\} \Rightarrow A = -2\ ;\ C = 1\ ;\ B = 8$$

SOLUCIÓN: $\boxed{\mathbf{I = L\left|\frac{4x^2+9}{x^2}\right| + \frac{1}{6}\,arc\,tg\,\frac{2x}{3} + C}}$

194. RESOLUCIÓN

$$I = \int \frac{x^3+1}{(x-1)^4}\,dx = 2\int \frac{dx}{(x-1)^4} + 3\int \frac{dx}{(x-1)^3} +$$

$$+ 3\int \frac{dx}{(x-1)^2} + \int \frac{dx}{x-1} =$$

$$= \frac{-2}{3\,(x-1)^3} - \frac{3}{2\,(x-1)^2} - \frac{3}{x-1} + L\,|x-1| + C$$

CÁLCULOS AUXILIARES

$$\frac{x^3+1}{(x-1)^4} = \frac{A}{(x-1)^4} + \frac{B}{(x-1)^3} + \frac{C}{(x-1)^2} + \frac{D}{x-1}$$

$$x^3 + 1 = A + B\,(x-1) + C\,(x-1)^2 + D\,(x-1)^3$$

$$x^3 + 1 = Dx^3 + (C-3D)\,x^2 + (B-2C+3D)\,x + A - B + C - D$$

$$\left.\begin{array}{l} 1 = D \\ 0 = C - 3D \\ 0 = B - 2C + 3D \\ 1 = A - B + C - D \end{array}\right\} \Rightarrow \left\{\begin{array}{l} A = 2 \\ B = 3 \\ C = 3 \\ D = 1 \end{array}\right.$$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{2}{3\,(x-1)^3} - \frac{3}{2\,(x-1)^2} - \frac{3}{x-1} + L\,|x-1| + C}}$

195. RESOLUCIÓN

$$I = \int \frac{dx}{x^4-13x^2+36} = -\frac{1}{30}\int \frac{dx}{x+3} + \frac{1}{30}\int \frac{dx}{x-3} +$$

$$+ \frac{1}{20}\int \frac{dx}{x+2} - \frac{1}{20}\int \frac{dx}{x-2} =$$

$$= -\frac{1}{30}\,L\,|x+3| + \frac{1}{30}\,L\,|x-3| + \frac{1}{20}\,L\,|x+2| -$$

$$- \frac{1}{20}\,L\,|x-2| + C = \frac{1}{30}\,L\left|\frac{x-3}{x+3}\right| + \frac{1}{20}\,L\left|\frac{x+2}{x-2}\right| + C$$

CÁLCULOS AUXILIARES

$$\frac{1}{x^4-13x^2+36} = \frac{1}{(x+3)\,(x-3)\,(x+2)\,(x-2)} =$$

$$= \frac{A}{x+3} + \frac{B}{x-3} + \frac{C}{x+2} + \frac{D}{x-2}$$

$$1 = A\,(x-3)\,(x+2)\,(x-2) + B\,(x+3)\,(x+2)\,(x-2) +$$

$$+ C\,(x+3)\,(x-3)\,(x-2) + D\,(x+3)\,(x-3)\,(x+2)$$

para $x = -3 \Rightarrow 1 = -30A \Rightarrow A = -\frac{1}{30}$

para $x = 3 \Rightarrow 1 = 30B \Rightarrow B = \frac{1}{30}$

para $x = -2 \Rightarrow 1 = 20C \Rightarrow C = \frac{1}{20}$

para $x = 2 \Rightarrow 1 = -20D \Rightarrow D = -\frac{1}{20}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{30} L \left|\frac{x-3}{x+3}\right| + \frac{1}{20} L \left|\frac{x+2}{x-2}\right| + C}}$

196. RESOLUCIÓN

$$I = \int \frac{4x^2 + x + 1}{x^3 - 1}\,dx = 2\int \frac{dx}{x-1} + \int \frac{2x+1}{x^2+x+1}\,dx =$$

$$= 2\,L\,|x-1| + L\,|x^2+x+1| + C = L\,|(x-1)^2\,(x^2+x+1)| + C$$

$$\frac{4x^2+x+1}{x^3-1} = \frac{4x^2+x+1}{(x-1)\,(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$4x^2 + x + 1 = A\,(x^2+x+1) + (Bx+C)\,(x-1)$$

$$4x^2 + x + 1 = (A+B)\,x^2 + (A-B+C)\,x + A - C$$

$$\left.\begin{array}{l} 4 = A + B \\ 1 = A - B + C \\ 1 = A - C \end{array}\right\} \Rightarrow \begin{array}{l} A = 2 \\ B = 2 \\ C = 1 \end{array}$$

CÁLCULOS AUXILIARES

$x^3 - 1 = (x-1)\,(x^2+x+1)$

SOLUCIÓN: $\boxed{\mathbf{I = L\,|(x-1)^2\,(x^2+x+1)| + C}}$

197. RESOLUCIÓN

$$I = \int \frac{dx}{x^3+1} = \frac{1}{3}\int \frac{dx}{x+1} + \int \frac{-\frac{x}{3} + \frac{2}{3}}{x^2-x+1}\,dx =$$

$$= \frac{1}{3} L\,|x+1| + \frac{1}{3}\int \frac{-x+2}{x^2-x+1}\,dx = \frac{1}{3} L\,|x+1| -$$

$$- \frac{1}{3}\int \frac{x-2}{x^2-x+1}\,dx = \frac{1}{3} L\,|x+1| - \frac{1}{6}\int \frac{2x-4}{x^2-x+1}\,dx =$$

$$= \frac{1}{3} L\,|x+1| - \frac{1}{6}\int \frac{2x-4+3\cdot 3}{x^2-x+1}\,dx = \frac{1}{3} L\,|x+1| -$$

$$- \frac{1}{6}\int \frac{2x-1}{x^2-x+1}\,dx - \frac{1}{6}\int \frac{-3\,dx}{x^2-x+1} = \frac{1}{3} L\,|x+1| -$$

$$- \frac{1}{6} L\,|x^2-x+1| + \frac{1}{2}\int \frac{dx}{x^2-x+1} = \frac{1}{3} L\,|x+1| -$$

$$- \frac{1}{6} L\,|x^2-x+1| + \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= \frac{1}{3} L\,|x+1| - \frac{1}{6} L\,|x^2-x+1| +$$

$$+ \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}}\, \text{arc tg}\, \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} + C = \frac{1}{3} L\,|x+1| -$$

$$- \frac{1}{6} L\,|x^2-x+1| + \frac{\sqrt{3}}{3}\, \text{arc tg}\, \frac{2x-1}{\sqrt{3}} + C$$

CÁLCULOS AUXILIARES

$x^3 + 1 = (x+1)\,(x^2-x+1)$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{3} L\,|x+1| - \frac{1}{6} L\,|x^2-x+1| + \frac{\sqrt{3}}{3}\, arc\, tg\, \frac{2x-1}{\sqrt{3}} + C}}$

198. RESOLUCIÓN

$$I = \int \frac{dx}{x^3+8} = \frac{1}{12}\int \frac{dx}{x+2} + \int \frac{\left(-\frac{1}{12}x + \frac{1}{3}\right)dx}{x^2-2x+4} =$$

$$= \frac{1}{12} L\,|x+2| + \int \frac{\left(\frac{4-x}{12}\right)dx}{x^2-2x+4} =$$

$$= \frac{1}{12} L\,|x+2| + \frac{1}{12}\int \frac{4-x}{x^2-2x+4}\,dx =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{12}\int \frac{(x-4)\,dx}{x^2-2x+4} =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24}\int \frac{2x-8}{x^2-2x+4}\,dx =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24}\int \frac{2x-2-6}{x^2-2x+4}\,dx =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24}\left[\int \frac{(2x-2)\,dx}{x^2-2x+4} - 6\int \frac{dx}{x^2-2x+4}\right] =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24}\left[L\,|x^2-2x+4| - 6\int \frac{dx}{(x-1)^2+3}\right] =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24} L\,|x^2-2x+4| + \frac{6}{24}\int \frac{dx}{(x-1)^2+3} =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24} L\,|x^2-2x+4| + \frac{1}{4}\int \frac{\frac{dx}{3}}{\frac{(x-1)^2}{3}+1} =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24} L\,|x^2-2x+4| + \frac{1}{12}\int \frac{dx}{\left(\frac{x-1}{\sqrt{3}}\right)^2+1} =$$

$$= \frac{1}{12} L\,|x+2| - \frac{1}{24} L\,|x^2-2x+4| + \frac{\sqrt{3}}{12}\,arc\,tg\,\frac{x-1}{\sqrt{3}} + C$$

CÁLCULOS AUXILIARES

$$\frac{1}{x^3+8} = \frac{1}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$$

$$1 = A(x^2-2x+4) + (Bx+C)(x+2)$$

$$1 = (A+B)x^2 + (-2A+2B+C)x + 4A + 2C$$

$$\left.\begin{aligned} 0 &= A + B \\ 0 &= -2A + 2B + C \\ 1 &= 4A + 2C \end{aligned}\right\} \Rightarrow A = \frac{1}{12}\ ;\ B = -\frac{1}{12}\ ;\ C = \frac{1}{3}$$

SOLUCIÓN:

$$\boxed{\mathbf{I = \frac{1}{12} L\,|x+2| - \frac{1}{24} L\,|x^2-2x+4| + \frac{\sqrt{3}}{12}\,arc\,tg\,\frac{x-1}{\sqrt{3}} + C}}$$

199. RESOLUCIÓN

$$I = \int \frac{dx}{sen\,x} = \int \frac{\frac{2\,dt}{1+t^2}}{\frac{2t}{1+t^2}} = \int \frac{dt}{t} =$$

$$L\,|t| + C = L\left|tg\,\frac{x}{2}\right| + C$$

CÁLCULOS AUXILIARES

$$tg\,\frac{x}{2} = t\ ;\ sen\,x = \frac{2t}{1+t^2}$$

$$\frac{x}{2} = arc\,tg\,t$$

$$x = 2\,arc\,tg\,t$$

$$dx = \frac{2\,dt}{1+t^2}$$

SOLUCIÓN:

$$\boxed{\mathbf{I = L\left|tg\,\frac{x}{2}\right| + C}}$$

200. RESOLUCIÓN

$$= \int \frac{dx}{cos\,x} = \int \frac{\frac{2\,dt}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \int \frac{2\,dt}{1-t^2} =$$

$$= \int \left(\frac{1}{1+t} + \frac{1}{1-t} \right) dt = \int \frac{dt}{1+t} + \int \frac{dt}{1-t} =$$

$$= L\,|1+t| - L\,|1-t| + C = L \left| \frac{1+t}{1-t} \right| + C =$$

$$= L \left| \frac{1 + tg \frac{x}{2}}{1 - tg \frac{x}{2}} \right| + C$$

$$\int \frac{2\,dt}{1-t^2} = \int \frac{dt}{1+t} + \int \frac{dt}{1-t} = L\,|1+t| - L\,|1-t| =$$

$$= L \left| \frac{1+t}{1-t} \right|$$

$$\frac{2}{1-t^2} = \frac{A}{1+t} + \frac{B}{1-t} = \frac{A(1-t) + B(1+t)}{1-t^2}$$

$$2 = A(1-t) + B(1+t) = (-A+B)\,t + A + B$$

$$\left.\begin{array}{l} 0 = -A + B \\ 2 = A + B \end{array}\right\} \Rightarrow A = 1 \;;\; B = 1$$

CALCULOS AUXILIARES

$$tg \frac{x}{2} = t \;;\; dx = \frac{2\,dt}{1+t^2} \;;\; \cos x = \frac{1-t^2}{1+t^2}$$

SOLUCIÓN:

$$\mathbf{I = L \left| \frac{1 + tg \frac{x}{2}}{1 - tg \frac{x}{2}} \right| + C}$$

201. RESOLUCIÓN

$$I = \int \frac{dx}{1 + \operatorname{sen} x + \cos x} = \int \frac{\frac{2\,dt}{1+t^2}}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} =$$

$$= \int \frac{\frac{2\,dt}{1+t^2}}{\frac{1+t^2+2t+1-t^2}{1+t^2}} = \int \frac{2\,dt}{2+2t} = \int \frac{dt}{1+t} =$$

$$= L\,|1+t| + C = L \left| 1 + tg \frac{x}{2} \right| + C$$

CÁLCULOS AUXILIARES

$$tg \frac{x}{2} = t \;;\; \operatorname{sen} x = \frac{2t}{1+t^2} \;;\; \cos x = \frac{1-t^2}{1+t^2}$$

$$\frac{x}{2} = \operatorname{arc\,tg}\, t \;;\; x = 2 \operatorname{arc\,tg}\, t$$

$$dx = \frac{2\,dt}{1+t^2}$$

SOLUCIÓN:

$$\mathbf{I = L \left| 1 + tg \frac{x}{2} \right| + C}$$

202. RESOLUCIÓN

$$I = \int \frac{dx}{5 + 4\cos x} = \int \frac{\frac{2\,dt}{1+t^2}}{5 + 4 \cdot \frac{1-t^2}{1+t^2}} =$$

$$= \int \frac{\frac{2\,dt}{1+t^2}}{\frac{5+5t^2+4-4t^2}{1+t^2}} = \int \frac{2\,dt}{t^2+9} = 2\int \frac{3\,dz}{(3z)^2+9} =$$

$$\frac{6}{9}\int \frac{dz}{z^2+1} = \frac{2}{3} \operatorname{arc\,tg}\, z + C = \frac{2}{3} \operatorname{arc\,tg} \frac{t}{3} + C =$$

$$\frac{2}{3} \operatorname{arc\,tg} \frac{tg \frac{x}{2}}{3} + C$$

Faltan las soluciones correspondientes a las integrales 203 y 204.

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{2} L \left| arc\ tg \frac{x}{2} \right| - \frac{1}{4} arc\ tg^2 \frac{x}{2} + C}}$

205. RESOLUCIÓN

$$I = \int sen^3 x\, dx = \int sen^2 x \cdot sen\, x\, dx = \int (1 - cos^2 x)\, sen\, x\, dx =$$

$$= -\int (1 - t^2)\, dt = -\int dt + \int t^2\, dt = -t + \frac{t^3}{3} + C =$$

$$= -cos\, x + \frac{cos^3 x}{3} + C$$

CÁLCULOS AUXILIARES

$cos\, x = t$

$-sen\, x\, dx = dt$

$sen\, x\, dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = -cos\, x + \frac{cos^3 x}{3} + C}}$

206. RESOLUCIÓN

$$I = \int cos^3 x\, dx = \int cos^2 x \cdot cos\, x\, dx = \int (1 - sen^2 x)\, cos\, x\, dx =$$

$$= \int (1 - t^2)\, dt = \int dt - \int t^2\, dt = t - \frac{t^3}{3} + C =$$

$$= sen\, x - \frac{sen^3 x}{3} + C$$

CÁLCULOS AUXILIARES

$sen\, x = t$

$cos\, x\, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = sen\, x - \frac{sen^3 x}{3} + C}}$

207. RESOLUCIÓN

$$I = \int sen^5 x\, dx = \int sen^4 x \cdot sen\, x\, dx = \int (1 - cos^2 x)^2\, sen\, x\, dx =$$

$$= -\int (1 - t^2)^2\, dt = -\int (1 - 2t^2 + t^4)\, dt = -\int dt + 2\int t^2\, dt -$$

$$-\int t^4\, dt = -t + 2 \cdot \frac{t^3}{3} - \frac{t^5}{5} + C =$$

$$= -cos\, x + \frac{2\, cos^3 x}{3} - \frac{cos^5 x}{5} + C$$

CÁLCULOS AUXILIARES

$cos\, x = t$

$-sen\, x\, dx = dt$

$sen\, x\, dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = -cos\, x + \frac{2\, cos^3 x}{3} - \frac{cos^5 x}{5} + C}}$

208. RESOLUCIÓN

$$I = \int cos^5 x\, dx = \int cos^4 x \cdot cos\, x\, dx = \int (1 - sen^2 x)^2\, cos\, x\, dx =$$

$$= \int (1 - t^2)^2\, dt = \int (1 - 2t^2 + t^4)\, dt =$$

$$= \int dt - 2\int t^2\, dt + \int t^4\, dt =$$

$$= t - 2 \cdot \frac{t^3}{3} + \frac{t^5}{5} + C = sen\, x - \frac{2\, sen^3 x}{3} + \frac{sen^5 x}{5} + C$$

CÁLCULOS AUXILIARES

$sen\, x = t$

$cos\, x\, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = sen\, x - \frac{2\, sen^3 x}{3} + \frac{sen^5 x}{5} + C}}$

209. RESOLUCIÓN

$$= \int sen^4 x \cdot \cos x \, dx = \int t^4 \, dt = \frac{t^5}{5} + C = \frac{sen^5 x}{5} + C$$

CÁLCULOS AUXILIARES

$sen\ x = t$

$\cos x \, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{sen^5 x}{5} + C}}$

210. RESOLUCIÓN

$$I = \int \cos^2 x \cdot sen^3 x \, dx = \int \cos^2 x \cdot sen^2 x \cdot sen\ x \, dx =$$

$$= \int \cos^2 x \, (1 - \cos^2 x) \, sen\ x \, dx = - \int t^2 \, (1 - t^2) \, dt =$$

$$= - \int t^2 \, dt + \int t^4 \, dt = - \frac{t^3}{3} + \frac{t^5}{5} + C =$$

$$= - \frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

CÁLCULOS AUXILIARES

$\cos x = t$

$-sen\ x \, dx = dt$

$sen\ x \, dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = - \frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C}}$

211. RESOLUCIÓN

$$I = \int \frac{sen^3 x}{\cos^2 x} dx = \int \frac{sen^2 x \cdot sen\ x}{\cos^2 x} dx =$$

$$= \int \frac{(1 - \cos^2 x) \, sen\ x}{\cos^2 x} dx = - \int \frac{(1 - t^2) \, dt}{t^2} =$$

$$= - \int \frac{dt}{t^2} + \int dt = \frac{1}{t} + t + C = \frac{1}{\cos x} + \cos x + C$$

CÁLCULOS AUXILIARES

$\cos x = t$

$-sen\ x \, dx = dt$

$sen\ x \, dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{\cos x} + \cos x + C}}$

212. RESOLUCIÓN

$$I = \int \frac{\cos^3 x}{sen^2 x} dx = \int \frac{\cos^2 x \cdot \cos x}{sen^2 x} dx =$$

$$= \int \frac{(1 - sen^2 x) \cos x \, dx}{sen^2 x} = \int \frac{(1 - t^2) \, dt}{t^2} = \int \frac{dt}{t^2} - \int dt =$$

$$= - \frac{1}{t} - t + C = - \frac{1}{sen\ x} - sen\ x + C$$

CÁLCULOS AUXILIARES

$sen\ x = t$

$\cos x \, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = - \frac{1}{sen\ x} - sen\ x + C}}$

213. RESOLUCIÓN

$$I = \int \sqrt{sen\ x} \cdot \cos^3 x \, dx = \int \sqrt{sen\ x} \cdot \cos^2 x \cdot \cos x \, dx =$$

$$= \int \sqrt{sen\ x} \cdot (1 - sen^2 x) \cos x \, dx = \int \sqrt{t} \, (1 - t^2) \, dt =$$

$$= \int \sqrt{t} \, dt - \int \sqrt{t} \cdot t^2 \, dt = \int t^{1/2} \, dt - \int t^{5/2} \, dt =$$

$$= \frac{2}{3} t^{3/2} - \frac{2}{7} t^{7/2} + C = \frac{2 \, sen^{3/2} x}{3} - \frac{2 \, sen^{7/2} x}{7} + C$$

CÁLCULOS AUXILIARES

$sen\ x = t$

$\cos x \, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2\ sen^{3/2} x}{3} - \frac{2\ sen^{7/2} x}{7} + C}}$

214. RESOLUCIÓN

$$I = \int \frac{sen^3 x}{cos\ x}\,dx = \int \frac{sen^2 x \cdot sen\ x}{cos\ x}\,dx =$$

$$= \int \frac{(1 - cos^2 x)\, sen\ x}{cos\ x}\,dx = -\int \frac{(1 - t^2)}{t}\,dt = -\int \frac{dt}{t} +$$

$$+ \int t\,dt = -L\,|t| + \frac{t^2}{2} + C = -L\,|cos\ x| + \frac{cos^2 x}{2} + C$$

CÁLCULOS AUXILIARES

$cos\ x = t$

$-sen\ x\,dx = dt$

$sen\ x\,dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = -\,L\,|cos\ x| + \frac{cos^2 x}{2} + C}}$

215. RESOLUCIÓN

$$I = \int \frac{cos\ x}{sen^3 x}\,dx = \int \frac{dt}{t^3} = \int t^{-3}\,dt = \frac{t^{-2}}{-2} + C =$$

$$= -\frac{1}{2t^2} + C = -\frac{1}{2\,sen^2 x} + C$$

CÁLCULOS AUXILIARES

$sen\ x = t$

$cos\ x\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{1}{2\,sen^2\,x} + C}}$

216. RESOLUCIÓN

$$I = \int \frac{dx}{cos^4 x} = \int \frac{dx}{cos^2 x \cdot cos^2 x} = \int sec^2 x \cdot sec^2 x\,dx =$$

$$= \int (1 + tg^2 x)\, sec^2 x\,dx = \int (1 + t^2)\,dt = \int dt + \int t^2\,dt =$$

$$= t + \frac{t^3}{3} + C = tg\ x + \frac{tg^3 x}{3} + C$$

CÁLCULOS AUXILIARES

$sec^2 x = 1 + tg^2 x$

$tg\ x = t$

$sec^2 x\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = tg\ x + \frac{tg^3\,x}{3} + C}}$

217. RESOLUCIÓN

$$I = \int \frac{dx}{sen^4 x} = \int cosec^4 x\,dx = \int cosec^2 x \cdot cosec^2 x\,dx =$$

$$= \int (1 + ctg^2 x)\, cosec^2 x\,dx = -\int (1 + t^2)\,dt = -\int dt - \int t^2 dt =$$

$$= -t - \frac{t^3}{3} + C = -ctg\ x - \frac{ctg^3 x}{3} + C$$

CÁLCULOS AUXILIARES

$ctg\ x = t$

$-cosec^2 x\,dx = dt$

$cosec^2 x\,dx = -dt$

SOLUCIÓN: $\boxed{\mathbf{I = -ctg\ x - \frac{ctg^3\,x}{3} + C}}$

218. RESOLUCIÓN

$$I = \int sen^3 x \cdot cos^3 x\,dx = \int sen^3 x \cdot cos^2 x \cdot cos\ x\,dx =$$

$$= \int sen^3 x\,(1 - sen^2 x)\, cos\ x\,dx = \int t^3\,(1 - t^2)\,dt = \int t^3\,dt - \int t^5\,dt =$$

$$= \frac{t^4}{4} - \frac{t^6}{6} + C = \frac{sen^4 x}{4} - \frac{sen^6 x}{6} + C$$

CÁLCULOS AUXILIARES

$sen\ x = t$

$cos\ x\,dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{sen^4\, x}{4} - \frac{sen^6\, x}{6} + C}}$

219. RESOLUCIÓN

$$= \int cos^4 x\, dx = \int cos^2 x \cdot cos^2 x\, dx = \int \left(\frac{1 + cos\, 2x}{2}\right)^2 dx =$$

$$= \frac{1}{4}\int (1 + 2\, cos\, 2x + cos^2 2x)\, dx = \frac{1}{4}\int dx + \frac{1}{2}\int cos\, 2x +$$

$$+ \frac{1}{4}\int cos^2 2x\, dx = \frac{x}{4} + \frac{1}{2}\cdot\frac{sen\, 2x}{2} + \frac{1}{4}\int \frac{1 + cos\, 4x}{2} dx =$$

$$= \frac{x}{4} + \frac{sen\, 2x}{8} + \frac{x}{8} + \frac{sen\, 4x}{32} + C$$

CÁLCULOS AUXILIARES

$sen^2 x + cos^2 x = 1$

$cos^2 x - sen^2 x = cos\, 2x$

$2cos^2 x = 1 + cos\, 2x$

$$cos^2 x = \frac{1 + cos\, 2x}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{4} + \frac{sen\, 2\, x}{8} + \frac{x}{8} + \frac{sen\, 4\, x}{32} + C}}$

220. RESOLUCIÓN

$$I = \int tg^3 x\, dx = \int tg\, x \cdot tg^2 x\, dx = \int tg\, x\, (sec^2 x - 1)\, dx =$$

$$= \int tg\, x \cdot sec^2 x\, dx - \int tg\, x\, dx = \int t\, dt - \int tg\, x\, dx =$$

$$= \frac{t^2}{2} + L\,|cos\, x| + C = \frac{tg^2 x}{2} + L\,|cos\, x| + C$$

CÁLCULOS AUXILIARES

$tg\, x = t$

$sec^2 x\, dx = dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{tg^2\, x}{2} + L\,|cos\, x| + C}}$

221. RESOLUCIÓN

$$I = \int ctg^3 \frac{x}{3} dx = \int ctg\, \frac{x}{3} \cdot ctg^2 \frac{x}{3} dx =$$

$$= \int ctg\, \frac{x}{3}\left(cosec^2 \frac{x}{3} - 1\right) dx = \int ctg\, \frac{x}{3} \cdot cosec^2 \frac{x}{3} dx -$$

$$- \int ctg\, \frac{x}{3} dx = -3\int t\, dt - \int ctg\, \frac{x}{3} dx = -3 \cdot \frac{t^2}{2} -$$

$$- 3\, L\left|sen\, \frac{x}{3}\right| + C = -\frac{3\, ctg^2 \frac{x}{3}}{2} - 3\, L\left|sen\, \frac{x}{3}\right| + C$$

CÁLCULOS AUXILIARES

$$ctg\, \frac{x}{3} = t$$

$$-\frac{1}{3}\, cosec^2 \frac{x}{3}\, dx = dt$$

$$cosec^2 \frac{x}{3}\, dx = -3\, dt$$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{3\, ctg^2 \frac{x}{3}}{2} - 3\, L\left|sen\, \frac{x}{3}\right| + C}}$

222. RESOLUCIÓN

$$I = \int sec^4 2x\, dx = \int sec^2 2x \cdot sec^2 2x\, dx =$$

$$= \int (1 + tg^2 2x)\, sec^2 2x\, dx = \int sec^2 2x\, dx + \int tg^2 2x \cdot sec^2 2x\, dx =$$

$$= \frac{1}{2}\, tg\, 2x + \int t^2 \frac{dt}{2} = \frac{1}{2}\, tg\, 2x + \frac{3}{2}\cdot t^3 + C =$$

$$= \frac{tg\, 2x}{2} + \frac{3\, tg^3 2x}{2} + C$$

CÁLCULOS AUXILIARES

$tg\, 2x = t$

$2\, sec^2 2x\, dx = dt$

$$sec^2 2x\, dx = \frac{dt}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{tg\ 2x}{2} + \frac{3\ tg^3\ 2x}{2} + C}}$

223. RESOLUCIÓN

$$I = \int sen\ 3x \cdot sen\ 2x\ dx = \frac{1}{2}\int (cos\ x - cos\ 5x)\ dx =$$

$$= \frac{1}{2}\int cos\ x\ dx - \frac{1}{2}\int cos\ 5x\ dx = \frac{1}{2} sen\ x -$$

$$- \frac{1}{2} \cdot \frac{sen\ 5x}{5} + C = \frac{sen\ x}{2} - \frac{sen\ 5x}{10} + C$$

CÁLCULOS AUXILIARES

$$sen\ a \cdot sen\ b = \frac{1}{2}[cos\ (a - b) - cos\ (a + b)]$$

$a = 3x\ ;\ b = 2x$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{sen\ x}{2} - \frac{sen\ 5x}{10} + C}}$

224. RESOLUCIÓN

$$I = \int sen\ 4x \cdot cos\ 2x\ dx = \frac{1}{2}\int (sen\ 6x + sen\ 2x)\ dx =$$

$$= \frac{1}{2}\int sen\ 6x\ dx + \frac{1}{2}\int sen\ 2x\ dx =$$

$$= \frac{1}{2} \cdot \frac{-cos\ 6x}{6} + \frac{1}{2} \cdot \frac{-cos\ 2x}{2} + C$$

$$= - \frac{cos\ 6x}{12} - \frac{cos\ 2x}{4} + C$$

CÁLCULOS AUXILIARES

$$sen\ a \cdot cos\ b = \frac{1}{2}[sen\ (a + b) + sen\ (a - b)]$$

$a = 4x\ ;\ b = 2x$

SOLUCIÓN: $\boxed{\mathbf{I = -\frac{cos\ 6x}{12} - \frac{cos\ 2x}{4} + C}}$

225. RESOLUCIÓN

$$I = \int cos\ 4x \cdot cos\ 3x\ dx = \frac{1}{2}\int (cos\ 7x + cos\ x)\ dx =$$

$$= \frac{1}{2}\int cos\ 7x\ dx + \frac{1}{2}\int cos\ x\ dx =$$

$$= \frac{1}{2} \cdot \frac{sen\ 7x}{7} + \frac{1}{2} \cdot sen\ x + C$$

CÁLCULOS AUXILIARES

$$cos\ a \cdot cos\ b = \frac{1}{2}[cos\ (a + b) + cos\ (a - b)]$$

$a = 4x\ ;\ b = 3x$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{sen\ 7x}{14} + \frac{sen\ x}{2} + C}}$

226. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = \int \frac{6t^5\ dt}{t^3 + t^2} = 6\int \frac{t^3}{t + 1}\ dt =$$

$$= 6\int \left(t^2 - t + 1 - \frac{1}{1 + t}\right) dt = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - L\,|1 + t|\right] + C =$$

$$= 2t^3 - 3t^2 + 6t - 6\,L\,|1 + t| + C =$$

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\,L\,|1 + \sqrt[6]{x}| + C$$

CÁLCULOS AUXILIARES

$m.c.m.\ (2, 3) = 6$

$x = t^6\ ;\ dx = 6t^5\,dt$

$$\begin{array}{r|l} -t^3 & t + 1 \\ \hline -t^3 - t^2 & t^2 - t + 1 \\ \hline -t^2 & \\ \hline t^2 + t & \\ \hline t & \\ \hline -t - 1 & \\ \hline -1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6L\,|1 + \sqrt[6]{x}| + C}}$

227. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt[4]{x^3} - \sqrt{x}} = \int \frac{4t^3\,dt}{t^3 - t^2} = 4\int \frac{t}{t-1}\,dt =$$

$$= 4\int \left(1 + \frac{1}{t-1}\right)dt = 4\,[t + L\,|t-1|] + C =$$

$$= 4t + 4\,L\,|t-1| + C = 4\sqrt[4]{x} + 4\,L\,|\sqrt[4]{x} - 1| + C$$

CÁLCULOS AUXILIARES

$m.c.m.\ (4, 2) = 4$

$x = t^4\ ;\ dx = 4t^3\,dt$

$$\begin{array}{r|l} t & t-1 \\ \hline -t+1 & 1 \\ \hline 1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = 4\sqrt[4]{x} + 4L\,|\sqrt[4]{x} - 1| + C}}$

228. RESOLUCIÓN

$$I = \int \frac{x^{1/4}}{1 + x^{1/2}}\,dx = \int \frac{t}{1+t^2} \cdot 4t^3\,dt = 4\int \frac{t^4}{1+t^2}\,dt =$$

$$= 4\int \left(t^2 - 1 + \frac{1}{1+t^2}\right)dt = 4\left[\frac{t^3}{3} - t + \text{arc tg } t\right] + C =$$

$$= 4\left[\frac{\sqrt[4]{x}}{3} - \sqrt[4]{x} + \text{arc tg } \sqrt[4]{x}\right] + C$$

CÁLCULOS AUXILIARES

$m.c.m.\ (4, 2) = 4$

$x = t^4\ ;\ dx = 4t^3\,dt$

$$\begin{array}{r|l} t^4 & t^2+1 \\ \hline -t^4 - t^2 & t^2 - 1 \\ \hline -t^2 & \\ \hline t^2 + 1 & \\ \hline 1 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = 4\left[\frac{\sqrt[4]{x}}{3} - \sqrt[4]{x} + \text{arc tg } \sqrt[4]{x}\right] + C}}$

229. RESOLUCIÓN

$$I = \int \frac{\sqrt{x} - \sqrt[6]{x}}{\sqrt[3]{x} + 1} = \int \frac{t^3 - t}{t^2 + 1} \cdot 6t^5\,dt = 6\int \frac{t^8 - t^6}{t^2+1}\,dt =$$

$$= 6\int \left(t^6 - 2t^4 + 2t^2 - 2 + \frac{2}{t^2+1}\right)dt =$$

$$= 6\left[\frac{t^7}{7} - 2\frac{t^5}{5} + 2\frac{t^3}{3} - 2t + 2\,\text{arc tg } t\right] + C =$$

$$= 6\left[\frac{\sqrt[6]{x^7}}{7} - \frac{2\sqrt[6]{x^5}}{5} + \frac{2\sqrt[6]{x^3}}{3} - 2\sqrt[6]{x} + 2\,\text{arc tg } \sqrt[6]{x}\right] + C$$

CÁLCULOS AUXILIARES

$m.c.m.\ (2, 6, 3) = 6$

$x = t^6\ ;\ dx = 6t^5\,dt$

$$\begin{array}{r|l} t^8 - t^6 & t^2+1 \\ \hline -t^8 - t^6 & t^6 - 2t^4 + 2t^2 - 2 \\ \hline -2t^6 & \\ \hline 2t^6 + 2t^4 & \\ \hline 2t^4 & \\ \hline -2t^4 - 2t^2 & \\ \hline -2t^2 & \\ \hline 2t^2 + 2 & \\ \hline 2 & \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = 6\left[\frac{\sqrt[6]{x^7}}{7} - \frac{2\sqrt[6]{x^5}}{5} + \frac{2\sqrt[6]{x^3}}{3} - 2\sqrt[6]{x} + 2\,\text{arc tg } \sqrt[6]{x}\right] + C}}$

230. RESOLUCIÓN

$$I = \int \frac{x^5\,dx}{\sqrt{x^3-1}} = \int \frac{x^3}{\sqrt{x^3-1}} \cdot x^2\,dx = \int \frac{t^2+1}{t} \cdot \frac{2}{3} t\,dt =$$

$$= \frac{2}{3}\int (t^2+1)\,dt = \frac{2}{3}\left[\frac{t^3}{3} + t\right] + C =$$

$$= \frac{2}{3}\left[(\sqrt{x^3-1})^3 + \sqrt{x^3-1}\right] + C$$

CÁLCULOS AUXILIARES

$\sqrt{x^3-1} = t \;;\; x^3 - 1 = t^2$

$3x^2\,dx = 2t\,dt$

$x^2\,dx = \frac{2}{3}\,t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2}{3}\left[(\sqrt{x^3-1})^3 + \sqrt{x^3-1}\right] + C}}$

231. RESOLUCIÓN

$$I = \int \frac{(2+x)\,dx}{\sqrt{x+3}} = \int \frac{(2+t^2-3)}{t} \cdot 2t\,dt = 2\int (t^2-1)\,dt =$$

$$= 2\left[\frac{t^3}{3} - t\right] + C = 2\,\frac{t^3}{3} - 2t + C = \frac{2x\,(x+3)^{1/2}}{3} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x+3} = t \;;\; x + 3 = t^2$

$x = t^2 - 3 \;;\; dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{2x\,(x+3)^{1/2}}{3} + C}}$

232. RESOLUCIÓN

$$I = \int \frac{dx}{x\sqrt{x-1}} = \int \frac{2t\,dt}{(t^2+1)\,t} = 2\int \frac{dt}{1+t^2} =$$

$$= 2 \text{ arc tg } t + C = 2 \text{ arc tg } \sqrt{x-1} + C$$

CÁLCULOS AUXILIARES

$\sqrt{x-1} = t$

$x - 1 = t^2 \;;\; x = t^2 + 1 \;;\; dx = 2t\,dt$

SOLUCIÓN: $\boxed{\mathbf{I = 2 \text{ arc tg } \sqrt{x-1} + C}}$

233. RESOLUCIÓN

$$I = \int \frac{(x+3)\,dx}{(x+5)\sqrt{x+4}} = \int \frac{(t^2-4+3)\,2t\,dt}{(t^2-4+5)\,t} =$$

$$= \int \frac{(t^2-1)\,2t\,dt}{(t^2+1)\,t} = 2\int \frac{t^2-1}{t^2+1}\,dt = 2\int \left(1 - \frac{2}{1+t^2}\right) dt =$$

$$= 2\,[t - 2 \text{ arc tg } t] + C = 2\,[\sqrt{x+4} - 2 \text{ arc tg } \sqrt{x+4}] + C$$

CÁLCULOS AUXILIARES

$\sqrt{x+4} = t$

$x + 4 = t^2 \;;\; x = t^2 - 4$

$dx = 2t\,dt$

$$\begin{array}{r} t^2 - 1 \\ -t^2 - 1 \\ \hline -2 \end{array} \qquad \Big\lfloor \begin{array}{l} t^2 + 1 \\ \hline 1 \end{array}$$

SOLUCIÓN: $\boxed{\mathbf{I = 2\,[\sqrt{x+4} - 2 \text{ arc tg } \sqrt{x+4}] + C}}$

234. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{1+\sqrt{1+x}}} = \int \frac{2t\,dt}{\sqrt{1+t}} = \int \frac{2\,(z^2-1)\,2z\,dz}{z} =$$

$$= 4\int (z^2-1)\,dz = 4\left[\frac{z^3}{3} - z\right] + C =$$

$$= \frac{4z^3 - 12z}{3} + C = \frac{z\,(4z^2-12)}{3} + C =$$

$$= \frac{\sqrt{1+t}\,[4\,(1+t)-12]}{3} + C = \frac{\sqrt{1+t}\,(4t-8)}{3} + C =$$

$$= \frac{\sqrt{1+\sqrt{1+x}}}{3} \cdot [4\sqrt{1+x} - 8] + C$$

CÁLCULOS AUXILIARES

$\sqrt{1+x} = t$

$1 + x = t^2$

$dx = 2t\,dt$

$\sqrt{1+t} = z$

$1 + t = z^2 \;;\; t = z^2 - 1$

$dt = 2z\,dz$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{\sqrt{1+\sqrt{1+x}}}{3}\,[4\sqrt{1+x} - 8] + C}}$

235. RESOLUCIÓN

$$I = \int \sqrt{1-x^2}\,dx = \int \sqrt{1-sen^2\,t} \cdot cos\,t\,dt = \int cos^2\,t\,dt =$$

$$= \frac{1}{2}\int (1 + cos\,2t)\,dt = \frac{1}{2}\int dt + \frac{1}{2}\int cos\,2t\,dt =$$

$$= \frac{1}{2}\,t + \frac{1}{4}\,sen\,2t + C = \frac{t}{2} + \frac{1}{4}\,2\,sen\,t\,cos\,t + C =$$

$$= \frac{arc\,sen\,x}{2} + \frac{1}{2}\,x\,\sqrt{1-x^2} + C$$

CÁLCULOS AUXILIARES

$x = sen\,t \;;\; t = arc\,sen\,x$

$dx = cos\,t\,dt$

$cos^2\,t + sen^2\,t = 1$

$cos^2\,t - sen^2\,t = cos\,2t$

$2\,cos^2\,t = 1 + cos\,2t$

$$cos^2\,t = \frac{1 + cos\,2t}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{arc\,sen\,x}{2} + \frac{1}{2}\,x\sqrt{1-x^2} + C}}$

236. RESOLUCIÓN

$$I = \int \sqrt{4-x^2}\,dx = \int \sqrt{4 - 4\,sen^2\,t} \cdot 2\,cos\,t\,dt =$$

$$= 4\int \sqrt{1-sen^2\,t} \cdot cos\,t\,dt = 4\int cos^2\,t\,dt =$$

$$= 4\int \frac{1 + cos\,2t}{2}\,dt = 2\int dt + 2\int cos\,2t\,dt =$$

$$= 2t + 2 \cdot \frac{1}{2}\,sen\,2t + C = 2t + sen\,2t + C =$$

$$= 2\,arc\,sen\,\frac{x}{2} + 2\,sen\,t \cdot cos\,t + C =$$

$$= 2\,arc\,sen\,\frac{x}{2} + \frac{x}{2}\,\sqrt{4-x^2} + C$$

CÁLCULOS AUXILIARES

$x = 2\,sen\,t \;;\; dx = 2\,cos\,t\,dt$

$$sen\,t = \frac{x}{2} \Rightarrow t = arc\,sen\,\frac{x}{2}$$

$$sen\,2t = 2\,sen\,t\,cos\,t = x\sqrt{1-sen^2\,t} = \frac{x}{2}\,\sqrt{4-x^2}$$

SOLUCIÓN: $\boxed{\mathbf{I = 2\,arc\,sen\,\frac{x}{2} + \frac{x}{2}\,\sqrt{4-x^2} + C}}$

237. RESOLUCIÓN

$$I = \int \sqrt{25-9x^2}\,dx = \int \sqrt{25-(3x)^2}\,dx =$$

$$= \int \sqrt{25-25\,sen^2\,t} \cdot \frac{5}{3}\,cos\,t\,dt = \frac{25}{3}\int \sqrt{1-sen^2\,t} \cdot cos\,t\,dt =$$

$$= \frac{25}{3}\int cos^2\,t\,dt = \frac{25}{3}\int \frac{1 + cos\,2t}{2}\,dt =$$

$$= \frac{25}{6}\int dt + \frac{25}{6}\int \cos 2t\, dt = \frac{25}{6}t + \frac{25}{12}\operatorname{sen} 2t + C =$$

$$= \frac{25}{6}\operatorname{arc\,sen}\frac{3x}{5} + \frac{25}{12}\cdot\frac{6x}{25}\sqrt{25 - 9x^2} + C =$$

$$= \frac{25}{6}\operatorname{arc\,sen}\frac{3x}{5} + \frac{x}{2}\sqrt{25 - 9x^2} + C$$

CÁLCULOS AUXILIARES

$3x = 5 \operatorname{sen} t$

$3\,dx = 5 \cos t\, dt$

$dx = \frac{5}{3}\cos t\, dt$

$\operatorname{sen} t = \frac{3x}{5}$

$t = \operatorname{arc\,sen}\frac{3x}{5}$

$$\operatorname{sen} 2t = 2 \operatorname{sen} t \cos t = 2\cdot\frac{3x}{5}\sqrt{1 - \frac{9x^2}{25}} = \frac{6x}{25}\sqrt{25 - 9x^2}$$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{25}{6}\operatorname{arc\,sen}\frac{3x}{5} + \frac{x}{2}\sqrt{25 - 9x^2} + C}}$

238. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{25 - 9x^2}} = \int \frac{\frac{5}{3}\cos t\, dt}{\sqrt{25 - 25\operatorname{sen}^2 t}} =$$

$$= \frac{1}{3}\int \frac{\cos t\, dt}{\sqrt{1 - \operatorname{sen}^2 t}} = \frac{1}{3}\int \frac{\cos t}{\cos t}dt = \frac{1}{3}\int dt =$$

$$= \frac{1}{3}t + C = \frac{1}{3}\operatorname{arc\,sen}\frac{3x}{5} + C$$

CÁLCULOS AUXILIARES

$3x = 5 \operatorname{sen} t$

$3\,dx = 5 \cos t$

$dx = \frac{5}{3}\cos t\, dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{1}{3}\operatorname{arc\,sen}\frac{3x}{5} + C}}$

239. RESOLUCIÓN

$$I = \int \frac{x\, dx}{\sqrt{x^2 - 4}} = \int \frac{2\sec t \cdot 2\sec t \cdot \operatorname{tg} t\, dt}{\sqrt{4\sec^2 t - 4}} =$$

$$= \frac{4}{2}\int \frac{\sec^2 t \cdot \operatorname{tg} t}{\sqrt{\sec^2 t - 1}}dt = 2\int \frac{\sec^2 t \cdot \operatorname{tg} t}{\operatorname{tg} t}dt =$$

$$= 2\int \sec^2 t\, dt = 2\operatorname{tg} t + C = 2\sqrt{\sec^2 t - 1} + C =$$

$$= 2\sqrt{\frac{x^2}{4} - 1} + C = \sqrt{x^2 - 4} + C$$

CÁLCULOS AUXILIARES

$x = 2\sec t \;;\; \sec t = \frac{x}{2}$

$dx = 2\sec t \cdot \operatorname{tg} t\, dt$

$\operatorname{tg}^2 t = \sec^2 t - 1$

SOLUCIÓN: $\boxed{\mathbf{I = \sqrt{x^2 - 4} + C}}$

240. RESOLUCIÓN

$$I = \int \sqrt{1 + 9x^2}\, dx = \int \sqrt{1 + (3x)^2}\, dx =$$

$$= \int \sqrt{1 + \operatorname{tg}^2 t}\cdot\frac{\sec^2 t\, dt}{3} = \frac{1}{3}\int \sec t \cdot \sec^2 t\, dt =$$

$$= \frac{1}{3}\int \sec^3 t\, dt = \frac{1}{3}\cdot\frac{1}{2}[\sec x \operatorname{tg} x + L\,|\sec x + \operatorname{tg} x|] + C =$$

$$= \frac{1}{6}[\sqrt{1 + 9x^2}\cdot 3x + L\,|\sqrt{1 + 9x^2} + 3x|] + C =$$

$$= \frac{x}{2}\sqrt{1 + 9x^2} + \frac{1}{6}L\,|3x + \sqrt{1 + 9x^2}| + C$$

ÁLCULOS AUXILIARES

$x = tg\ t$

$dx = sec^2 t\ dt$

$x = \dfrac{sec^2 t\ dt}{3}$

$+ tg^2 t = sec^2 t$

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SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{2}\sqrt{1 + 9x^2} + \frac{1}{6} L\,|3x + \sqrt{1 + 9x^2}| + C}}$

11. RESOLUCIÓN

$$= \int \frac{\sqrt{x^2 - 1}}{x} dx = \int \frac{\sqrt{sec^2 t - 1}}{sec\ t} \cdot sec\ t \cdot tg\ t\ dt =$$

$$\int \sqrt{sec^2 t - 1} \cdot tg\ t\ dt = \int tg\ t \cdot tg\ t\ dt = \int tg^2 t\ dt =$$

$$\int (tg^2 t + 1 - 1)\ dt = \int sec^2 t\ dt - \int dt = tg\ t - t + C =$$

$$\sqrt{sec^2 t - 1} - t + C = \sqrt{x^2 - 1} - arc\ sen\ x + C$$

ÁLCULOS AUXILIARES

$= sec\ t\ ;\ t = arc\ sec\ x$

$x = sec\ t \cdot tg\ t\ dt$

$+ tg^2 t = sec^2 t$

$^2 t = sec^2 t - 1$

SOLUCIÓN: $\boxed{\mathbf{I = \sqrt{x^2 - 1} - arc\ sen\ x + C}}$

12. RESOLUCIÓN

$$= \int \sqrt{8x - x^2}\, dx = \int \sqrt{16 - (x - 4)^2}\, dx =$$

$$\int \sqrt{16 - 16\ sen^2 t} \cdot 4\ cos\ t\ dt = 16 \int \sqrt{1 - sen^2 t} \cdot cos\ t\ dt =$$

$$16 \int cos^2 t\ dt = 16 \int \frac{1 + cos\ 2t}{2} dt = 8 \int (1 + cos\ 2t)\ dt =$$

$$= 8 \int dt + 8 \int cos\ 2t\ dt = 8t + 8 \cdot \frac{sen\ 2t}{2} + C =$$

$$= 8\ arc\ sen\ \frac{x - 4}{4} + \frac{x - 4}{2} \sqrt{8x - x^2} + C$$

CÁLCULOS AUXILIARES

$x - 4 = 4\ sen\ t\ ;\ dx = 4\ cos\ t\ dt$

SOLUCIÓN: $\boxed{\mathbf{I = 8\ arc\ sen\ \frac{x - 4}{4} + \frac{x - 4}{2} \cdot \sqrt{8x - x^2} + C}}$

243. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{(x^2 + a^2)^3}} = \int \frac{a\ sec^2 t\ dt}{\sqrt{(a^2\ tg^2 t + a^2)^3}} =$$

$$= a \int \frac{sec^2 t\ dt}{\sqrt{[a^2\ (tg^2 t + 1)]^3}} = a \int \frac{sec^2 t\ dt}{\sqrt{(a^2 \cdot sec^2 t)^3}} =$$

$$= \frac{a}{a^3} \int \frac{sec^2 t\ dt}{sec^3 t} = \frac{1}{a^2} \int \frac{dt}{sec\ t} = \frac{1}{a^2} \int cos\ t\ dt =$$

$$= \frac{1}{a^2}\ sen\ t + C = \frac{1}{a^2} \cdot \frac{x}{\sqrt{x^2 + a^2}} + C$$

CÁLCULOS AUXILIARES

$x = a\ tg\ t$

$dx = a\ sec^2 t\ dt$

$\sqrt{x^2 + a^2} = a\ sec\ t$

$tg\ t = \dfrac{x}{a}$

$sen\ t = \dfrac{x}{\sqrt{x^2 + a^2}}$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{a^2 \sqrt{x^2 + a^2}} + C}}$

244. RESOLUCIÓN

$$I = \int \frac{x^2\,dx}{\sqrt{(1-x^2)^3}} = \int \frac{sen^2 t \cdot cos\, t\, dt}{\sqrt{(1-sen^2 t)^3}} = \int \frac{sen^2 t \cdot cos\, t\, dt}{\sqrt{(cos^2 t)^3}} =$$

$$= \int \frac{sen^2 t \cdot cos\, t\, dt}{cos^3 t} = \int \frac{sen^2 t}{cos^2 t} dt = \int tg^2 t\, dt =$$

$$= \int (tg^2 t + 1 - 1)\, dt = \int (tg^2 t + 1)\, dt - \int dt =$$

$$= \int sec^2 t\, dt - \int dt = tg\, t - t + C = \frac{sen\, t}{cos\, t} - t + C =$$

$$= \frac{x}{\sqrt{1-x^2}} - arc\, sen\, x + C$$

CÁLCULOS AUXILIARES

$x = sen\, t\ ;\ t = arc\, sen\, x$

$dx = cos\, t\, dt$

SOLUCIÓN: $\boxed{\mathbf{I = \frac{x}{\sqrt{1-x^2}} - arc\, sen\, x + C}}$

245. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{x^2+1}\,(\sqrt{x^2+1}+x)} = \int \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}\,(x^2+1-x^2)} dx =$$

$$= \int \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}} dx = \int \left(1 - \frac{x}{\sqrt{x^2+1}}\right) dx =$$

$$= \int dx - \int \frac{x\,dx}{\sqrt{x^2+1}} = \int dx - \frac{1}{2}\int \frac{2x\,dx}{\sqrt{x^2+1}} =$$

$$= x - \frac{1}{2} \cdot 2\sqrt{x^2+1} + C = x - \sqrt{x^2+1} + C$$

SOLUCIÓN: $\boxed{\mathbf{I = x - \sqrt{x^2+1} + C}}$

246. RESOLUCIÓN

$$I = \int \sqrt{3-2x-x^2}\, dx = \int \sqrt{4-(x+1)^2} \cdot dx =$$

$$= \int \sqrt{4-4\,sen^2 t} \cdot 2\,cos\, t\, dt = 4\int \sqrt{1-sen^2 t} \cdot cos\, t\, dt =$$

$$= 4\int cos\, t \cdot cos\, t\, dt = 4\int cos^2 t\, dt = 4\int \frac{1+cos\, 2t}{2} dt =$$

$$= 2\int dt + 2\int cos\, 2t\, dt = 2t + sen\, 2t + C =$$

$$= 2t + 2\, sen\, t\, cos\, t + C =$$

$$= 2\, arc\, sen\, \frac{x+1}{2} + 2 \cdot \frac{x-1}{2} \cdot \sqrt{1 - \frac{(x+1)^2}{4}} + C =$$

$$= 2\, arc\, sen\, \frac{x+1}{2} + \frac{x-1}{2} \cdot \sqrt{4-(x+1)^2} + C$$

CÁLCULOS AUXILIARES

$x + 1 = 2\, sen\, t\ ;\ dx = 2\, cos\, t\, dt$

$$sen\, t = \frac{x+1}{2}$$

$$t = arc\, sen\, \frac{x+1}{2}$$

SOLUCIÓN: $\boxed{\mathbf{I = 2\, arc\, sen\, \frac{x+1}{2} + \frac{x-1}{2} \cdot \sqrt{4-(x+1)^2} + C}}$

247. RESOLUCIÓN

$$I = \int \frac{dx}{\sqrt{4\,ax - x^2}} = \int \frac{dx}{\sqrt{4a^2-(x-2a)^2}} =$$

$$= \int \frac{2a\,cos\, t\, dt}{\sqrt{4a^2-4a^2\,sen^2 t}} = \frac{2a}{2a}\int \frac{cos\, t}{\sqrt{1-sen^2 t}} dt =$$

$$= \int \frac{cos\, t}{cos\, t} dt = \int dt = t + C = arc\, sen\, \frac{x-a}{2a} + C$$

CÁLCULOS AUXILIARES

$x - a = 2a \operatorname{sen} t$

$dx = 2a \cos t \, dt$

$$\operatorname{sen} t = \frac{x - a}{2a}$$

$$t = \operatorname{arc\,sen} \frac{x - a}{2a}$$

SOLUCIÓN: $$\boxed{\mathbf{I = \operatorname{arc\,sen} \frac{x - a}{2a} + C}}$$

248. RESOLUCIÓN

$$I = \int \frac{x \, dx}{4 - x^2 + \sqrt{4 - x^2}} = \int \frac{2 \operatorname{sen} t \cdot 2 \cos t \, dt}{4 - 4 \operatorname{sen}^2 t + \sqrt{4 - 4 \operatorname{sen}^2 t}} =$$

$$= \int \frac{4 \operatorname{sen} t \cos t \, dt}{4 (1 - \operatorname{sen}^2 t) + \sqrt{4 (1 - \operatorname{sen}^2 t)}} = \int \frac{4 \operatorname{sen} t \cos t \, dt}{4 \cos^2 t + 2 \cos t} =$$

$$= \int \frac{4 \operatorname{sen} t \, dt}{4 \cos t + 2} = \int \frac{\operatorname{sen} t}{\cos t + \frac{1}{2}} dt = \int \frac{-dz}{z} =$$

$$= -L|z| + C = -L\left|\cos t + \frac{1}{2}\right| + C = -L\left|\sqrt{1 - \operatorname{sen}^2 t} + \frac{1}{2}\right| + C =$$

$$= -L\left|\sqrt{1 - \frac{x^2}{4}} + \frac{1}{2}\right| + C = -L\left|\frac{\sqrt{4 - x^2} + 1}{2}\right| + C$$

CÁLCULOS AUXILIARES

$x = 2 \operatorname{sen} t \;;\; \operatorname{sen} t = \frac{x}{2}$

$dx = 2 \cos t \, dt$

$\cos t + \frac{1}{2} = z$

$-\operatorname{sen} t \, dt = dz$

$\operatorname{sen} t \, dt = -dz$

SOLUCIÓN: $$\boxed{\mathbf{I = -L\left|\frac{\sqrt{4 - x^2} + 1}{2}\right| + C}}$$